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2. The Theory of Special Relativity

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2. The Theory of Special Relativity

Abstract
Albert Einstein (1879-1955) published his first work on relativity in 1905, the same year in which he published remarkable papers on Brownian motion and the photoelectric effect. At the time he did this work, he was a patent examiner in the Swiss Patent Office. He was awarded the Nobel Prize for physics in 1921 "for his services to the theory of physics, and especially for his discovery of the law of the photoelectric effect." He became a professor of physics at several German universities, and in 1916, he took a position at the Kaiser Wilhelm Institute in Berlin.

As the Nazi party became powerful and finally took control of the country, Einstein became a target of the Nazi's anti-Jewish campaign. He left Germany with regret and found sanctuary in the United States. In 1933 he became a permanent staff member at the Institute for Advanced Studies at Princeton. He remained at that post for the rest of his life.

Einstein proposed a solution to the puzzle posed by the Michelson-Morley results, and that work has come to be known as the theory of special relativity. Einstein's solution came as a surprise to most physicists because it was based not upon some strange new principle, but upon two postulates that would have been conceded by nearly all and upon a careful scrutiny of some accepted concepts. [*excerpt*]

Keywords
Contemporary Civilization, Einstein, Relativity, Special Relativity, Physics

Disciplines
History of Science, Technology, and Medicine | Physics

Comments
This is a part of Section XX: Meaning in the Physical Sciences. The Contemporary Civilization page lists all additional sections of *Ideas and Institutions of Western Man,* as well as the Table of Contents for both volumes.

More About Contemporary Civilization:

From 1947 through 1969, all first-year Gettysburg College students took a two-semester course called Contemporary Civilization. The course was developed at President Henry W.A. Hanson's request with the goal of "introducing the student to the backgrounds of contemporary social problems through the major concepts, ideals, hopes and motivations of western culture since the Middle Ages."

Gettysburg College professors from the history, philosophy, and religion departments developed a textbook for the course. The first edition, published in 1955, was called *An Introduction to Contemporary Civilization and Its Problems*. A second edition, retitled *Ideas and Institutions of Western Man*, was published in 1958 and 1960. It is this second edition that we include here. The copy we digitized is from the Gary T. Hawbaker '66 Collection and the marginalia are his.

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2. The Theory of Special Relativity

Albert Einstein (1879-1955) published his first work on relativity in 1905, the same year in which he published remarkable papers on Brownian motion and the photoelectric effect. At the time he did this work, he was a patent examiner in the Swiss Patent Office. He was awarded the Nobel Prize for physics in 1921 "for his services to the theory of physics, and especially for his discovery of the law of the photoelectric effect." He became a professor of physics at several German universities, and in 1916, he took a position at the Kaiser Wilhelm Institute in Berlin.

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Einstein sees these in a fresh outlook.

The two postulates are:

(1) The velocity of light in a vacuum is the same in all coordinate systems that move with constant velocity relative to each other.

(2) All laws of nature are the same in all coordinate systems that move with constant velocity relative to each other.

The first of these is simply the acceptance of the results of the Michelson-Morley experiment. The second was not completely new with Einstein, Newton having made the same statement with reference to mechanical laws. The new statement means that no experiment of any kind (including electromagnetic experiments) can tell us whether we are at rest or moving with constant velocity, since the very form of our mathematical equations expressing our physical laws must remain the same in all systems with constant relative velocity. This is often referred to as the condition of invariance.

A simple thought-experiment will give us an idea of the way Einstein was thinking and show us how he brought some of our long-standing concepts into serious question.

Imagine a room in a moving train, a lamp in the center of the room, and on one side wall a window large enough to allow someone outside the train to see the entire room. Now imagine that as the train passes a man standing on the embankment, another man on the train turns the lamp on and off quickly. We ask the two men to describe what they see.

The man on the train says that the light traveling from the center of the room with equal velocity in all directions reaches the front and back walls of the room simultaneously, since the walls are equidistant from the lamp.

The man on the embankment agrees that the light travels with the same velocity in all directions. Further, he agrees with the man on the train as to the velocity of the light. We surely expected this from the first of Einstein's postulates, which is just the Michelson-Morley result. But the man on the embankment says further that while the light was traveling from the lamp to the walls, the front wall was trying to get away from the light and the back wall was rushing to meet the light. Thus he observed that the light reached the back wall before it reached the front wall. The situation as seen by the two men is shown in Figure IV.

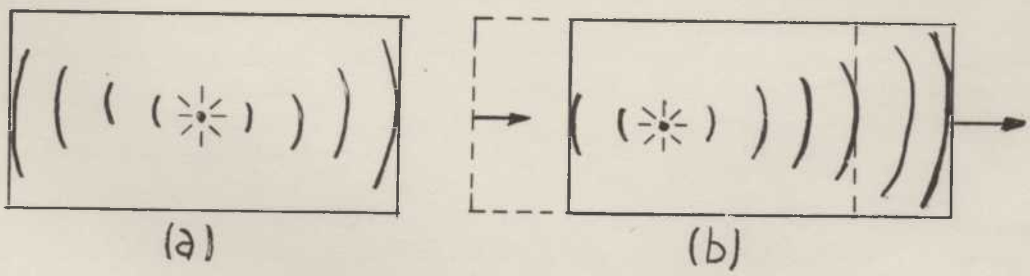

Figure IV. Two views of a light pulse: (a) as seen by the man in the train, and (b) as seen by the man on the embankment.

The results of our thought-experiment may seem innocuous enough, but if we examine them we will discover something startling. What the man on the train observed to be simultaneous, the man on the embankment observed to be not simultaneous. Now, who is right? Did the light reach the front and back walls of the room simultaneously, or did it not? Before we leap to answer the question, we should look carefully at what we mean by the words "simultaneous," "sooner," and "later."

These words had an absolute meaning when our equations taking us from one coordinate system to another were the Galilean transformations. What appeared to be simultaneous events in one coordinate system would be inferred to be simultaneous in all coordinate systems. By "inferred" we mean the following. Suppose the two events were light flashes and that there were some measured time interval between our seeing the two flashes. If we could now measure the distance to each event, then knowing the velocity of light we could determine whether the two events occurred simultaneously. If we concluded that the two events were simultaneous, then if the Galilean transformations were valid, we would be certain that anyone else who saw the flashes would also infer that the flashes occurred simultaneously. But our thought-experiment with men in and out of the train might well make us suspicious of the concept of simultaneity as derived from the Galilean transformations.

The following is essentially the way Einstein approached the problem. First we ask, "What is a clock?" Einstein answered, "We understand by a clock something which provides a series of events which can be counted." Any physical system that provides an occurrence that can be repeated exactly may be used as a clock. We can take the interval between the start and the end of the occurrence as the unit of time. By counting the number of occurrences in our standard system, we can measure time intervals and associate the words "sooner" and "later" with the smaller and larger numbers registered by the clock. The earth's motion provides us with two clocks. The time associated with one rotation about its axis is the day, and the time associated with the interval required for the earth to make one circuit about the sun is the year. Even an hourglass fits our description, since by counting the number of times the glass is turned we can measure the elapsed time in hours. Today we have clocks based upon particular vibrations that occur in certain molecules.

Suppose that we have two clocks at different locations in some coordinate system. How can we be certain that these clocks are synchronized, in other words that they are showing exactly the same time and that they are running at the same rate? If we are at different distances from the two clocks, then even synchronized clocks would appear to read different times, since light would take a longer time reaching us from one of the two. This difficulty is overcome simply, if we stand at a point equidistant from the two clocks. Then if the clocks always show the same time, we can use them to designate the times at which events occur at the two clock locations. We now repeat this process, putting clocks at as many points as we care to. Since we are only doing this in our imaginations at present, we might as well put a clock at every point in our coordinate system. We are thus assured that all of our clocks are at rest in our coordinate system and that they are all synchronized. The time at which an event occurs in our system will be given by the clock located at the position at which the event occurs. We can stand in one place in our coordinate system and take note of an event and the time of its occurrence without making any correction for the velocity of light, in other words for the time light takes to reach us from the event.

We have done nothing that does violence to the Galilean transformations. We have simply made things more convenient than would be the case were we to have but one clock which would necessitate corrections for our distances from that clock and from the event we were observing. We have been quite careful though, and our care may even appear to be excessive at this point. One might ask, "Why did you not synchronize all your clocks at two very nearby points, making a collection of synchronized clocks, and then distribute them to the various points in your coordinate system?" We reply that we do not know what effect motion would have on the rate at which the clocks run, so we avoid any possible oversight by placing our clocks as we have described.

In a completely similar way, we can put clocks in all coordinate systems which move at constant velocity relative to the first system. We are assured, by our definition of simultaneity and by our procedure for placing the clocks, that all the clocks in a given coordinate system are synchronized. We now ask, "Are the clocks in one of these systems synchronized with those in another?"

Newton would have answered "Yes" unhesitatingly. In fact, he would be certain that a single clock would suffice for both systems, for he believed and stated that "Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external..." (see Chapter VIII, p. 58). But Newton did not know of the Michelson-Morley experiment, and his prediction would have been based upon the Galilean transformation. But we have seen that these two, the Michelson-Morley result and the Galilean transformations, cannot be reconciled. Further, we have seen that our thought-experiment about the train suggests that the idea of simultaneity may be more subtle than Newton had believed. We should not close our minds to the possibility that the clocks in two coordinate systems moving with constant velocity relative to each other may not be synchronized, in other words that they may not be running at the same rate. Possibly the frequency at which our clock mechanisms operate would be different when the clocks have different velocities relative to an observer.

With the Newtonian concept of simultaneity brought into question, other concepts must be reexamined carefully. For example, how do we measure the length of an object? Imagine an object to be at rest in our coordinate system. We note the points in this coordinate system that coincide with the ends of the object, and then we measure the distance between these two points. For this we have a rod of defined length (one yard one meter, etc.), and we count the number of times we can lay this rod end to end along the straight line connecting the two points. The number we get is defined as the length of the object in whatever units we choose. As we shall see, this process may be very complex and itself requires considerable analysis, but we shall not examine the details here.

We now ask, "How shall we measure the length of an object that is moving relative to us?" We must mark the positions of the ends of the object in our coordinate system, but these positions must be marked simultaneously. We can do this, if we have synchronized clocks at these two points. We then measure the distance between these points in our coordinate system in the manner prescribed above.

Now, if we measure the length of an object both when it is at rest and when it is in motion relative to our coordinate system, do we get the same number for the length in each case? Newton would have replied, "Yes." But we should be careful at this date not to give a quick reply to questions such as this on the basis of what we might call common sense. There is nothing in the world to tell us how to answer this question without making a careful experimental analysis. And even then, we should be careful to note that our reply may have validity only in the range covered by our experiments.

Another example arises in measuring the mass of an object. The mass of an object is proportional to its weight at a given location on the earth's surface. Further, the body with the greater mass exhibits a greater resistance to a change in its velocity. The greater the mass of an object, the greater the force required to change the object's velocity by a given amount in a given time. This latter property that mass measures is part of Newton's second law of motion. We can ask, in fact we must ask, "Is the mass of an object the same when we make our measurements at two different velocities relative to the object?" Again, Newton would have answered "Yes."

Einstein was able to bring order into the theory. He saw clearly that the Newtonian concepts of absolute time, length, and mass, manifested in the Galilean transformations, were simply at odds with experimental results. The ideas of absolute time and space can have meaning to us only if we can know at what absolute time an event occurs and at what absolute position an object is located. And it is clear that we do not know these things, and our theory (Newtonian mechanics as well as relativity) has built-in conditions which make it impossible to know these things. We do, in fact, measure time intervals, as Einstein has stated, by counting the number of times some regular event occurs. And we do not locate our coordinate systems in some abstract space, but we do locate them relative to some material body or bodies: the earth, the solar system, our galaxy -- the Milky Way.

If we wanted to insist that time, length, and mass are independent of relative velocities, then we would need to define these quantities in some way quite different from the way they are now defined. If we do not insist upon this requirement for our theory, we shall be able to avoid the necessity of constructing a different theory for each coordinate system, which would surely be the ultimate relativity. But in order to avoid this thoroughly distasteful alternative, Einstein found that the Galilean transformations must be discarded. If we are going to use the definitions of time and length given by Einstein and still satisfy the two postulates of relativity, then the transformations required are just those proposed by Lorentz. But now those transformations are no longer a disconnected mathematical artifice; they are a necessary part of a complete theory that encompasses all of physics, and not merely a single phenomenon.

The Lorentz transformations give results that are sometimes surprising to those who have not given serious thought to the concepts used in recent physical theories. The first concerns time, and says that the clocks synchronized in one coordinate system do not run at the same rate as clocks synchronized in another system that is moving with constant velocity relative to the first. In mathematical form, where c is the velocity of light in a vacuum (186,000 miles per second)

$$\Delta t' = \Delta t \sqrt{1 - V^2/c^2}$$

similar to P. 14

If we are at rest in our coordinate system and our clocks have moved through the time interval Δt, then we shall observe that the clocks at rest in a coordinate system that is moving with a constant velocity, V, relative to us will show the corresponding interval $\Delta t'$. Note that $\Delta t'$ is always less than Δt; that is, we observe that moving clocks are running slow. Figure V illustrates this point.

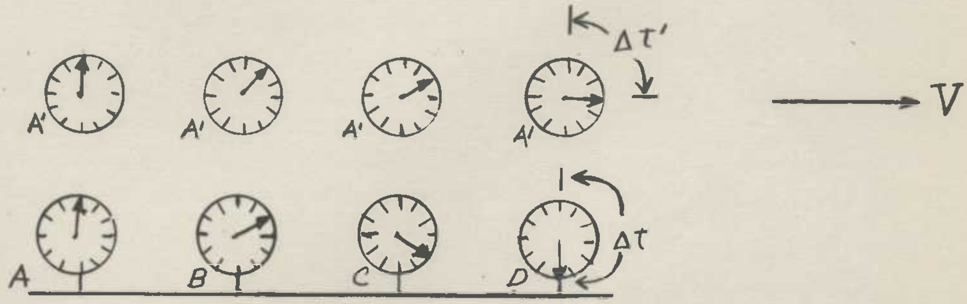

Figure V. The Lorentz transformation predicts that we observe moving clocks to be running slow.

If we observe the single clock A' moving with a velocity V relative to our clocks, we see that A' is running slow. By using many clocks in our system, we are assured that our observations of A and A', B and A', C and A', etc., are simultaneous. Further, we note that our caution in not moving our clocks in our coordinate system once they were synchronized is justified. If $\Delta t'$ were just half of Δt, as shown in the figure, the velocity V would need to be about 86.5 percent of the velocity of light, or about 161,000 miles per second.

It is quite important to note that someone at rest in a coordinate system having some velocity relative to our own would observe that our clocks are running slow. To ask which of the clocks are running at the correct rate makes no sense.

as $V \to c$ $\sqrt{\ }$ → fraction comes closer to one.

We also note that $\Delta t'$ differs from Δt significantly only when the relative velocity V is very high. If V is about ten percent of the velocity of light, or about 18,600 miles per second, then $\Delta t'$ differs from Δt by just one percent. In the case that V is zero, the Lorentz transformation reduces to $\Delta t' = \Delta t$.

The Lorentz transformation for length is

$$L = L_o\sqrt{1 - V^2/c^2}$$

where L_o is the length we would measure were the object at rest relative to us, and L is the length we would measure were the object moving with a velocity V relative to us. Here the length L is being measured along the same line as the velocity V is directed. Note that as V increases, the object's length in the direction of V decreases. Figure VI illustrates this change.

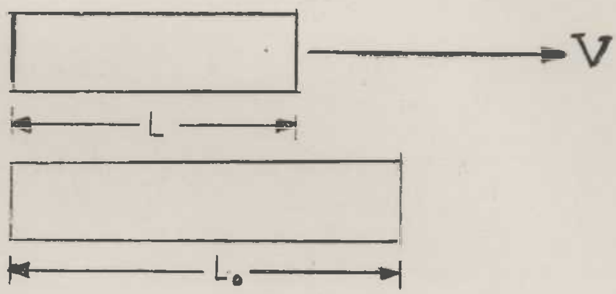

__Figure__ __VI__. The Lorentz transformation predicts that we will observe rods to become shorter when they are moving relative to us.

The Lorentz transformation for mass is

$$m = \frac{m_o}{\sqrt{1 - V^2/c^2}}$$

where m_o and m are the masses we measure when the object is at rest and moving with a velocity V relative to us respectively. The mass increases with increasing V.

We can see from the last two transformations (length and mass) that an object's velocity relative to any observer must be less than the velocity of light. We note that as the velocity of a body increases, the body's mass increases also, so that greater and greater forces are required to produce velocity changes when the velocity is close to that of light. The two transformations would predict that at V = c, an object would have zero length (and hence zero volume) and infinite mass. But we could not expect to reach this velocity, since an infinite force would be required.

If the consequences of the Lorentz transformations are examined in detail, we find that the theory handles the concepts of mass and energy in completely equivalent ways. This is the basis for the famous equation $E = mc^2$, where E and m are the mass and energy of a system. The conservation laws of energy and mass are combined into a single energy-mass conservation law. If a certain amount of mass Δm disappears in some process, then an associated amount of energy ΔE appears, where $E = (\Delta m) c^2$. Just this mass loss accounts for the energy that appears in atomic and hydrogen bombs.

One further point should be made, this being probably the most important philosophical result of the theory. For the moment, let us restrict ourselves to observing events that occur on some chosen straight line. We shall call the space position along this line x, which is called positive when measured in one direction from a reference point (origin) and negative when measured in the other direction. Now suppose that we turn a light on and off in quick succession at the origin, so that two pulses of light travel away from the origin, one in the positive direction and one in the negative direction. We can make a simple plot that will show us the locations of the pulses at any time after they leave the origin. This plot is given in Figure VII.

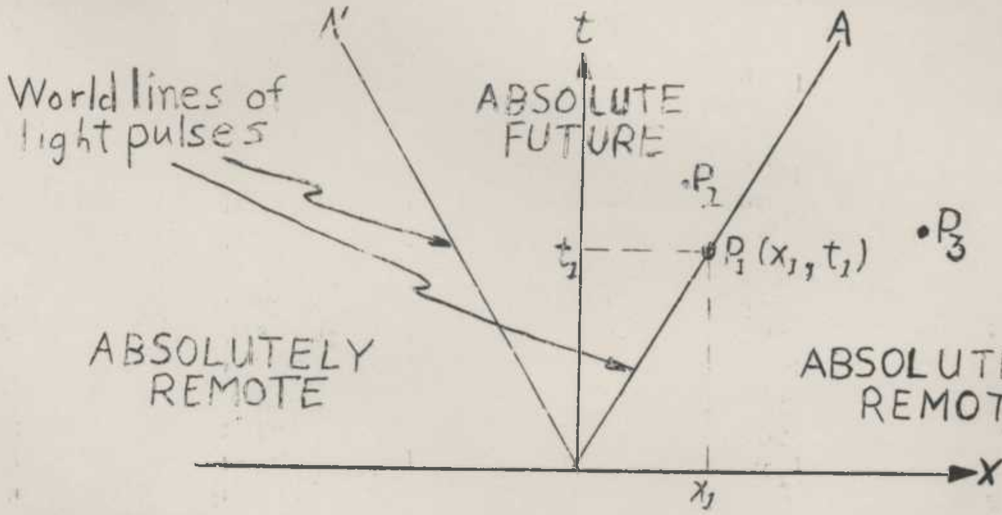

Figure VII. World lines of two light pulses which originated at x = O and t = O.

We plot the positions of the light pulses (x) against the time (t) at which the pulses arrive at those positions. For example, the light pulse is located at $x = x_1$ at the time $t = t_1$, so that the point P_1 (x_1, t_1) lies on one of our lines. This point might be $x_1 = 186{,}000$ miles and $t1 = 1$ second. The lines are straight, because the velocity of light is constant. That is, $x = ct$, so that $x/t = c$ at all points on the line. These two lines are called the world lines of the light pulses for our coordinate system, and they give a complete description of the behavior of the pulses both in time and in space. The world lines of all objects which pass through the origin O (that is,

Could there be a general concept drawn out of this. Suggestions of lessons that can be drawn from this.

are at $x = 0$ when $t = 0$) must lie only in the region AOA', since no object can have a velocity in any coordinate system which is greater than the velocity of light (c).

We shall say that an "event" is described by the space position where it occurred and the time when it occurred. In general, an event requires three space coordinates and the time for its description. These are the four dimensions that are often associated with relativity. In our example, where we are limiting ourselves to one space dimension, we have a two dimensional "space" called the space-time continuum. An event is represented by a point in that space, called a world point.

Consider now two world points, one at the origin O (0,0) and one at the point P_2 (x_2,t_2) lying in the region AOA'. The distance between these two events is simply x_2, and the time interval between them is t_2. Since P_2 lies in the region AOA', we see that ct_2 is greater than x2. That is, light leaving $x = 0$ at $t = 0$ would have reached the position x_2 before the second event (x_2,t_2) occurred. It turns out that there is no coordinate system, moving at any velocity whatever relative to our own, in which these two events occur simultaneously. Thus the time order of these two events is the same in all coordinate systems, [O(0,0) "before" $P_2(x_2,t_2)$], and we thus call the region AOA' the absolute future relative to O. There is, however, one coordinate system in which the two events will occur at the same place. The relativistic interval between two events of this kind is said to be timelike.

Consider now the two events O(0,0) and $P_3(x_3,t_3)$. Here the distance between the two events is x_3, and the time interval between them is t_3. Since P_3 lies outside AOA' we see that ct_3 is less than x_3. That is, light leaving $x = 0$ at $t = 0$ would arrive at the position x_3 after the second event (x_3,t_3) had occurred. Here there is another coordinate system in which the two events are simultaneous, and in fact there are an infinite number of coordinate systems in which the event (x_3,t_3) occurs before (0,0). Then we can assign no absolute time order to these events. There is, however, no coordinate system in which these two events occur at the same place. The relativistic interval between two events of this kind is said to be spacelike.

Since the time order of O(0,0) and $P_2(x_2,t_2)$ is absolute, it may be that there is a "causal" relation between them. Since the time order of O(0,0) and P_3 (x_3,t_3) depends upon the coordinate system in which the observer is at rest, we conclude that there can exist no such causal relation between these two events. Since the criterion for the relativistic interval between two events being timelike or spacelike is whether light starting from $x = 0$ at $t = 0$ reaches the position (x_2 or x_3) before or after the second event occurs, we conclude that no influence (force field) can have a propagation velocity greater than c. That is, no physical event can send out the signal of its occurrence with a velocity greater than c, the velocity of light.

We notice that the theory of special relativity tells us how to transform coordinates and time from one coordinate system to another moving with constant velocity relative to the first. The ghost of inertial systems haunts the special theory. Einstein could see no reason to give any preference to coordinate systems with uniform relative velocity, and he set about constructing a theory that would be generally applicable, even to accelerating coordinate systems. He published his first work on the theory of general relativity in 1915. We cannot discuss this theory in any detail here, but we can remark that general relativity, unmotivated by experiment, stands as one of the most extraordinary intellectual accomplishments in the history of man. Einstein, literally alone, wrought the theory with imagination, insight, and inspiration that may well have been unique. While the general theory is not in as common use as in the special theory, it not yet being required for the description of most physical phenomena, it has made some startling predictions which have been verified experimentally. No exception has yet been found to the general theory.

It is not surprising that the thoroughgoing success of relativity theory should send most serious thinkers in a variety of intellectual disciplines scurrying to retest their basic assumptions and definitions. We have yet to receive all the fruits that must follow such a reaction. Percy A. Bridgman (1882-) has long been one of those urging us to learn the lessons of relativity well. Bridgman was for many years a professor of physics at Harvard University, and he received the Nobel Prize in physics in 1946.

Bridgman advocates that definitions be based upon the operations, physical or mental, that we perform when actually using the definitions. Such definitions he calls operational. Also he argues for an open and receptive attitude while also insisting that we be cautious about extending our concepts into new realms in which they are not tested. In light of the latter point, the publication date of the following selection by Bridgman is significant. Within a short time after Bridgman wrote The Logic of Modern Physics in 1927, the experimental results in the atomic realm and their interpretation by the then new quantum mechanics were to again call into serious question almost all of the basic concepts on which the subject of physics was built. The new questions raised in that realm still are cause for disagreement and controversy among many of today's most eminent physicists.