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Abstract
With the U.S. presidential election fast approaching we will often be reminded that the candidate who receives the most votes is not necessarily elected president. Instead, the winning candidate must receive a majority of the 538 electoral votes awarded by the 50 states and the District of Columbia. Someone with a curious mathematical mind might then wonder: What is the small fraction of the popular vote a candidate can receive and still be elected president? [*excerpt*]

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270

How to Win the Presidency With Just 17.56% of the Popular Vote

CHUCK WESSELL

With the U.S. presidential election fast approaching we will often be reminded that the candidate who receives the most votes is not necessarily elected president. Instead, the winning candidate must receive a majority of the 538 electoral votes awarded by the 50 states and the District of Columbia. Someone with a curious mathematical mind might then wonder: What is the smallest fraction of the popular vote a candidate can receive and still be elected president?

In 1961 George Pólya, who certainly had a curious mathematical mind, considered exactly this question in a paper in he published in *The Mathematics Teacher*. Pólya's formulation of the question is an excellent example of how to simplify a real-world problem so it can be analyzed mathematically. Pólya's simplification involved making three assumptions:

- The number of votes cast in a state is *exactly* proportional to the number of that state's representatives in the U.S. Congress;
- There are two presidential candidates; and
- Each state gives all its electoral votes to the candidate with the largest number of popular votes in that state.

The mathematical argument Pólya made was in the context of the 1960 presidential election, and he concluded that a candidate could win that election with slightly more than 22 percent of the popular vote (see figure 1).

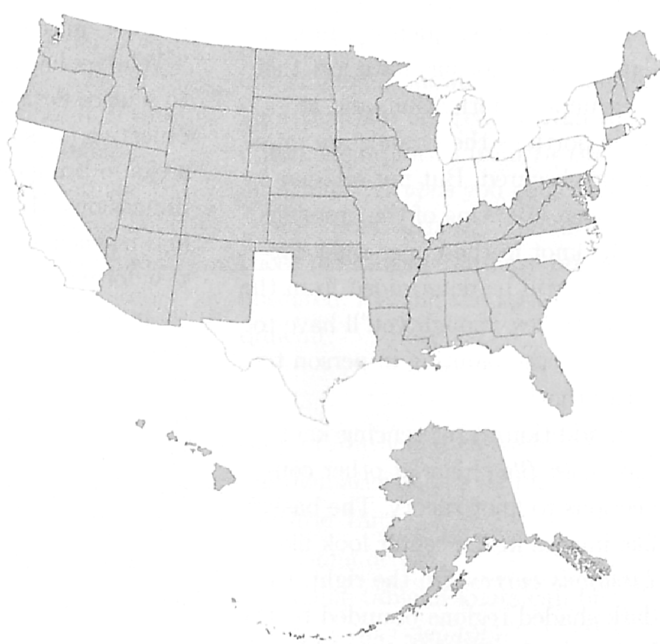

Figure 1. Pólya's 1960 solution estimated that by winning the 38 orange-shaded states by the smallest possible margin while receiving no votes in the unshaded states a candidate could be elected president with 22.1 percent of the popular vote. Using the actual vote totals from that election one can show that the 1960 winner could have received as little as 19.12 percent of the popular vote.

To apply Pólya's methodology to the 2012 election requires a slight reworking of his solution to take into account the 23rd Amendment, which gave electoral votes to the District of Columbia starting with the election of 1964. To account for this, we will use the word "state" to mean any of the 51 entities that award electoral votes, even though the District of Columbia is technically not a state. Additionally, the 1960 election had an unusual total of electoral votes because newly admitted states Alaska and Hawaii were each given three electoral votes, but no electoral votes were taken away from any of the other states. Despite these peculiarities, Pólya's method of solution remains both elegant and insightful.

Pólya's Solution (Updated for 2012)

Let v_i represent the number of votes cast in state i and r_i equal the number of U.S. representatives from that state. Then Pólya's first assumption says that there is a single proportionality constant k such that $v_i = kr_i$. Summing up the 51 equations of this form leads to

$$\sum_{i=1}^{51} v_i = k\sum_{i=1}^{51} r_i,$$

or

$$v = kr = k(436), \qquad (1)$$

where the unsubscripted v is the nationwide number of votes cast in the presidential election and $r = 436$ is the total number of state representatives (435) plus one "virtual" representative from the District of Columbia.

Since the number of electoral votes for a particular state is equal to its number of representatives plus two, a candidate is elected by winning states $s_1, s_2, \ldots, s_n$ as long as

$$\begin{aligned}(r_1+2)+(r_2+2)+\cdots+(r_n+2) &\geq 270, \text{ or} \\ r_1+r_2+\cdots+r_n &\geq 270-2n.\end{aligned} \qquad (2)$$

For simplicity of exposition, assume the total vote count $v_i = kr_i$ is *even* for each state. Then to win state i, at least $(kr_i/2)+1$ votes are needed. So the number of votes the winning candidate would receive from those n states is

$$\begin{aligned} w &\geq \left(\frac{kr_1}{2}+1\right)+\left(\frac{kr_2}{2}+1\right)+\cdots+\left(\frac{kr_n}{2}+1\right) \\ &= \frac{k}{2}(r_1+r_2+\cdots+r_n)+n. \end{aligned} \qquad (3)$$

If the winning candidate receives zero votes in the $(51-n)$ states not won, then the winner's fraction of the total vote count can be obtained by dividing (3) by (1).

$$\begin{aligned} \frac{w}{v} &\geq \frac{\frac{k}{2}(r_1+r_2+\cdots+r_n)+n}{436k} \\ &= \frac{(r_1+r_2+\cdots+r_n)}{872}+\frac{n}{436k}. \end{aligned}$$

Using (2) to replace the first numerator, we obtain

$$\frac{w}{v} \geq \frac{270-2n}{872}+\frac{n}{436k}. \qquad (4)$$

Minimizing the fraction w/v involves two steps. First, show that equality in (4) can hold, and then minimize the right-hand side of that equation.

Since (4) is obtained from earlier inequalities, equality will be obtained when both sides of those inequalities are equal. Notice that (2) is an equality if the candidate wins exactly 270 electoral votes, and (3) is an equality if the candidate wins those n states by the barest of margins (and receives no popular votes in any of the other states).

Turning now to minimizing the right-hand side of (4), first notice that the second fraction, $n/436k$, is equal to the number of states won divided by the popular vote for the entire country. Given that there are only 51 states to win and more than 130 million people voted in the 2008 election, this fraction's contribution to the final answer is negligible. Thus, the key to minimizing the winning candidate's proportion of the popular vote lies in minimizing the fraction $(270-2n)/872$. The fraction gets smaller as n, the number of states won, gets larger. The goal is to win exactly 270 electoral votes while winning as many states as possible. This means winning all or most of the states that have the fewest electoral votes.

The optimal combination of winning states is easily obtained for the 2012 election: Winning the 39 states with 12 electoral votes or fewer will give a candidate 255 electoral votes. Also winning the one state with 15 electoral votes (North Carolina) will result in exactly 270 electoral votes and $n = 40$ states won. (Pólya found $n = 38$ for the 1960 election. For every election since, the value of n has been either 39 or 40.)

Plugging $n = 40$ into (4) and using the 2008 vote total leads to

$$\frac{270-2(40)}{872}+\frac{40}{131{,}370{,}793} \approx 0.21789.$$

So, if a candidate won the 40 states shown in figure 2 by the barest of margins and received no other votes, that candidate would become president with about 21.8 percent of the popular vote. Moreover, under Pólya's assumptions, it is impossible to win the election with a lower percentage of the popular vote.

Anyone familiar with the political geography of the United States could rightfully argue that a candidate winning exactly the states featured in either the 1960 or 2012 solutions by a bare majority and also receiving zero votes in all other states is a near impossibility. Pólya admitted as much when he said such results would occur only "in some freak political constellation." Still, it is sobering to consider that current election laws allow for such a possibility.

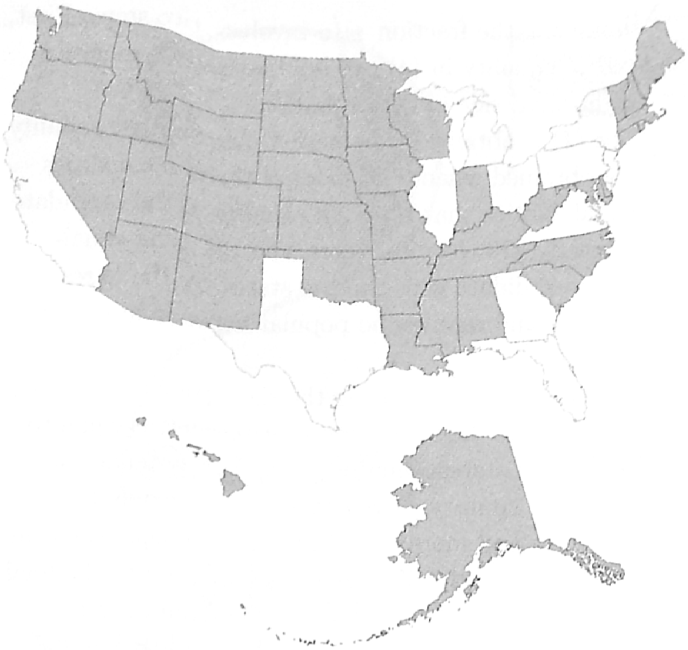

Figure 2. Pólya's solution for the upcoming presidential election requires winning the 39 orange-shaded states and the District of Columbia by the smallest possible margin while receiving no votes in the unshaded states. By doing so a candidate could be elected president with 21.8 percent of the popular vote.

Numerical Verification

Pólya's calculation of the minimal popular vote proportion required to win a presidential election is based on three assumptions. The assumption that departs furthest from reality is that there is a single proportionality constant k that relates the number of votes cast in a state to the number of its U.S. representatives. For example, in 2008 the proportionality "constant" for Hawaii was 226,784, compared with 491,092 for Montana. Though it greatly simplified calculations, this assumption ignores the fact that seats in the U.S. House of Representatives are apportioned using a state's total population, not the number of eligible, registered, or actual voters. Furthermore, apportionment is based on the most recent census, and some elections (for example, 2008) are held eight years later. You can probably think of other problems with this assumption.

What happens if we use the actual state vote counts v_i instead of kr_i? As stated earlier, the solution to the 1960 version of this problem entailed winning 38 states and 22.1 percent of the popular vote. Oddly, even though the actual state vote counts were readily available after the election, Pólya never compared his theoretical result to the popular vote percentage that would be obtained by winning those same 38 states (by the barest of margins and receiving no votes in other states) using the actual vote counts. If he had, he would have seen an even more incredible result: the possibility of a winning candidate who received only 19.12 percent of the votes cast. Once the 2012 election results are final, you can complete a similar exercise.

How Low Can You Go?

Today state-by-state vote totals for presidential elections are just a web search away. The data can easily be copied into a spreadsheet or read by a computer program. With that computing power at your disposal, you can begin a search for a solution with an even lower popular vote percentage for any particular presidential election. Because Pólya's argument is based on his three simplifying assumptions, a good place to look for improvement is where those assumptions and reality differ.

A trivial way to find a lower popular vote percentage is to allow there to be three candidates. Now a candidate needs only one vote more than one-third of those cast to win a particular state. This is intellectually unsatisfying since one could then just change the number of candidates to four, and then five, and so on. And of course in the upcoming presidential election, there are indeed two major party candidates.

Assumption three is almost a perfect match with reality. Only two states, Maine and Nebraska, do not award electoral votes on a winner-take-all basis. These states award one electoral vote to the winner in each congressional district and the remaining two electoral votes to the winner of the entire state. Finding presidential vote totals broken down by congressional district can be a challenge, so exploiting this assumption to find a better solution will be left to the most serious political junkies.

As mentioned earlier, the assumption that departs the most from reality is that there is a single proportionality constant that relates the number of votes cast in a state to the number of its U.S. representatives. We have seen that, using actual state-by-state vote totals, it is possible to find a scenario in which the winning candidate receives a lower percentage of the popular vote than Pólya's theoretical calculation. But how low can you go?

In the February 2012 issue of *Math Horizons*, Stan Wagon presented an introduction to integer linear programming (ILP). A subset of ILP in which each of the variables is either 0 or 1 is called *binary integer linear programming* (BILP). For this problem, a 1 is assigned to each state won and a 0 to each state lost. We want to find the set of zeros and ones such that the electoral vote sum of the "1 states" is at least 270 and the popular vote count is minimized. Using BILP, it is possible to find a way to win the 1960 election with just 17.56 percent of the vote (see figure 3).

If you have some ideas on how to improve on Pólya's result for a particular election, go to *http://www.maa.*

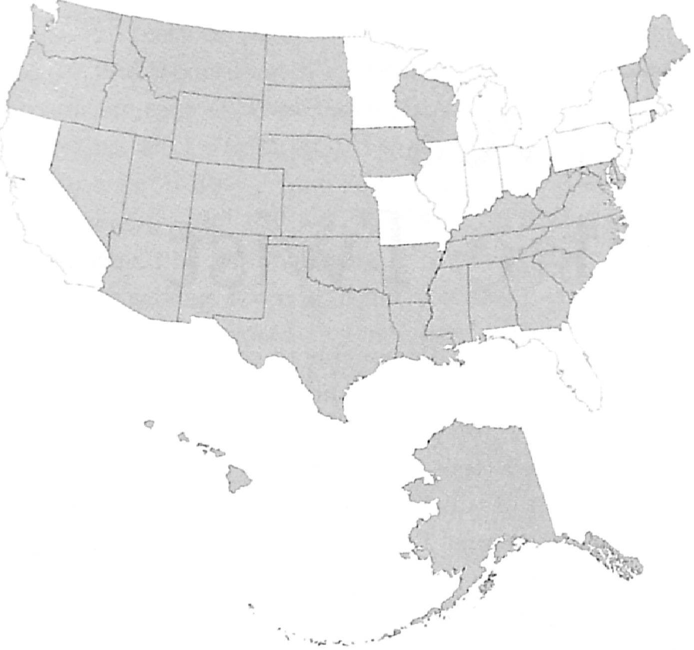

Figure 3. BILP solution to the minimum popular vote problem for the 1960 election. Winning the 37 orange-shaded states by the smallest possible margin while receiving no votes in the unshaded states, a candidate could be elected president with 17.56 percent of the popular vote.

org/mathhorizons/supplemental.htm. Here you will find a spreadsheet with state-by-state vote totals for all presidential elections back to 1960. ■

FURTHER READING

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J. Sullivan, "The election of a President," *The Mathematics Teacher*, October 1972, 493–501.

S. Wagon, "An algebraic approach to geometrical optimization," *Math Horizons*, February 2012, 22–27.

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http://dx.doi.org/10.4169/mathhorizons.20.1.18

The Geometry of Nature

CHAYA MORASHA GILBERT-MCNABB

I've tried to calculate the area
above the curve called planet Earth,
but somehow,
that mathematical constant eludes me.
They tell me to take slices
of an infinitesimal loaf of rye
but *Mandelbrot* is what satiates me.
As I knead another batch of dough,
the nuts make imprints on my knuckles.
No loaves I shape will ever be smooth,
for the reality I live is irregular.
Euclid tried to define me
with a compass and a straight-edge,
but I lie somewhere *between* the definitive
where knife and almond are in opposition.
I am the break-down of nature—
seventy facets, seven planes—
where each is multiplicable
by eight when it's asleep.
Imagination is my brush, infinity my paint;
unfettered dreams are realized on a finite page.
Some say I'm rough,
that twice-baked makes me tough,
but scaled up or down, I hold true
in both shape and dimension.
Like Elven waybread I sustain,
like Temple showbread I endure,
but Mandel bread is what I am:
My irregularity allows me
to walk the slippery slope of Earth.
Nothing I see is as it seems;
I look deeper and still grasp but a piece.
Perfections are exceptions;
iterations of life
aren't always differentiable.
You can plug in the past
to generate the future,
or integrate recursively
on an indefinite plane,
but I need not calculate beyond myself,
for I am
a fractal. ■

Chaya Morasha Gilbert-McNabb is a math student at Cal State, Los Angeles, who enjoys writing and gardening.