CAFE Symposium 2023 | Celebration of Academic First-Year Engagement

2-8-2023

The Use of Probability in Quantum Mechanics to Calculate Measurement Outcomes

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Abstract

The concept of probability can help measure some of the possible outcomes of different experiments in the field of quantum mechanics. Those experiments include Thomas Young's double slit experiment, the Schrödinger equation, the wave function, and the Born Rule, which all make use of probability to predict the placement of certain subatomic particles including photons of light, in the experiments. In this project, the manner in which probability does this is explored in depth.

Keywords

Probability, Quantum physics/mechanics, Double slit experiment, Wave functions, Laplace's demon

Disciplines

Other Physical Sciences and Mathematics | Probability

Comments

This poster was created based on work completed for FYS 146: Probability, Prediction, and Paradox; and presented as a part of the eighth annual CAFE Symposium on February 8, 2023.


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The use of Probability in Quantum Mechanics to Calculate Measurement Outcomes

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Background

-The discovery of classical physics, or classical mechanics can be recognized as the scientific explanations behind some things that we may do in our everyday lives such as wearing seatbelts, thanks to the idea of inertia and the concept of the conservation of momentum would explain why there is more of a kickback on stronger firearms.

-More of the basic principles of classical mechanics include ideas such as the law of universal gravitation, centripetal force, Newton's laws of motion, and much more.

-The main principle behind classical mechanics was that it was deterministic, that the future and the past are both determined by the present.

What is Quantum Mechanics?

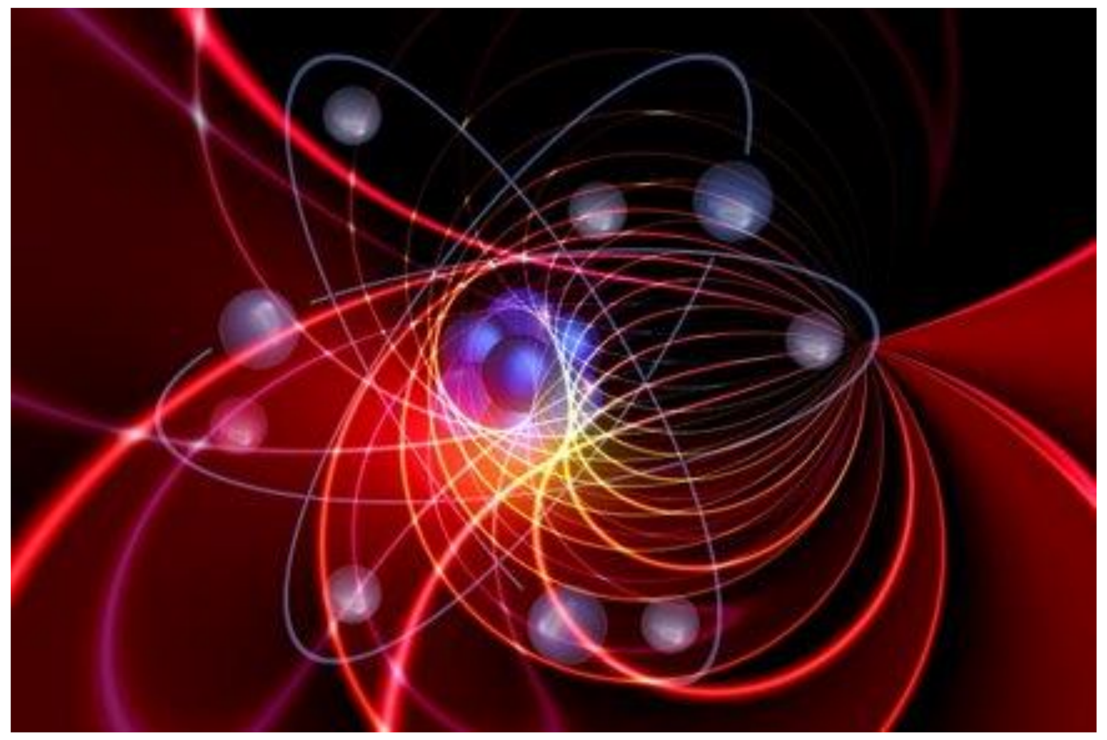
•Quantum mechanics emerged from the bulk of ideas, principles, and laws from classical mechanics nearly a century after it was discovered.

•This area of physics focuses on the behaviors of the building blocks of nature, such as photons and electrons, including the analysis of energy and matter. Quantum mechanics also analyzes how a certain system evolves over time.

•Neils Bohr, Max Planck, and Albert Einstein are a few scientists who are recognized for their groundbreaking work in the field of quantum mechanics. They are all recipients of multiple Nobel prizes. In fact, their work was so significant that they are known as the "founding fathers" of quantum mechanics.

•This field includes ideas such as the photoelectric effect or the emission of electrons when light hits a material, Einstein's photons of light, the wave-particle duality of light, the wave function, and how subatomic particles interact with both matter and energy.

•However, the concept that sets quantum mechanics apart from most other theories in physics is that it explains what a specific outcome is when a system is measured.


"Quantum Physics " is marked with CC0 1.0.

Objectives of this Research Project

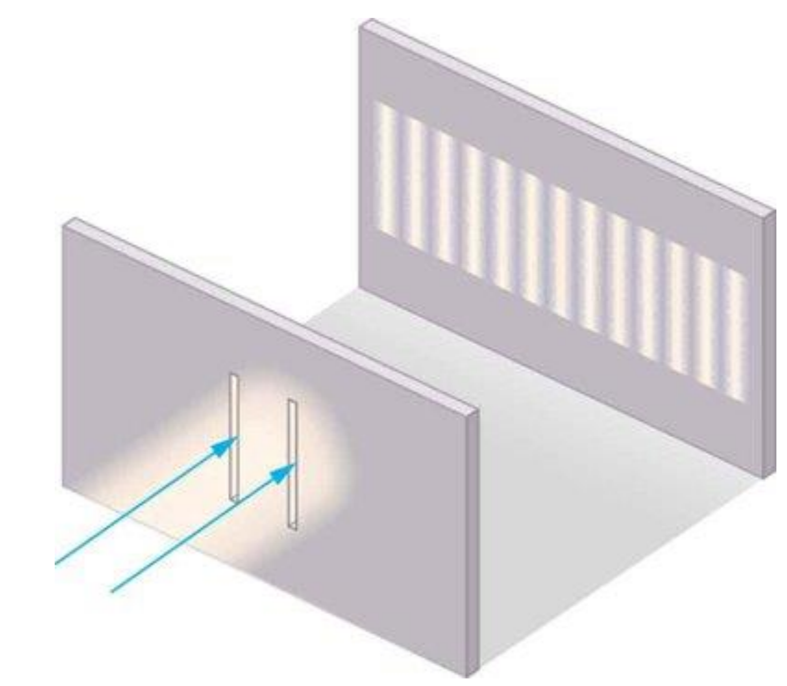
-Quantum mechanics can predict these specific outcomes, however, not with 100% confidence. Because of this, this area of physics makes use of probability to quantify each and every outcome.

-This use of probability in quantum mechanics is possible because underlining each theory in quantum mechanics, there are a series of mathematical equations that make use of probability to arrive at a prediction as to what the outcome may be for a certain problem.

-Each mathematical concept or equation is demonstrated by an experiment which shows how probability is used to measure the outcomes of these experiments in the field of quantum mechanics.

-Based on this, the ultimate objective of this research project was to explain how those outcomes were measured by demonstrating three examples of different experiments mentioned in this project that make use of probability to successfully quantify those experimental outcomes.

Experiment #1: Young's Double Slit Experiment


"Young's double slit experiment" by OpenStax is licensed under Creative Commons Attribution License v4.0

The Basics of this Procedure

-Thomas Young was a British physician and physicist, who, at the beginning of his career, studied medicine and set up a medical practice in London around the year of 1799.

-It was at this time, while he was focusing primarily on sense perception, that he would also begin to focus his studies on the behavior of light.

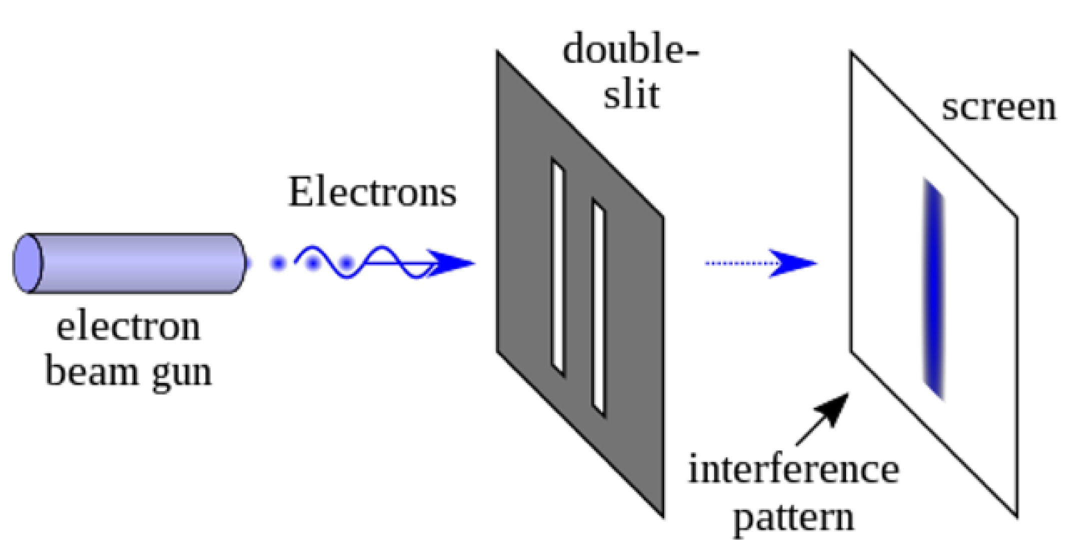
-Eventually, Young discovered certain principles of light and contributed to the theory of the wave-particle duality of light with his double-slit experiment.

-The theory of the wave particle duality illustrates that any subatomic particle or any quantum entity can be described not only as particles but also as waves.

-He began his experiment by letting light pass through two slits on a screen and when it did, he noticed that some of the beams of light overlapped and spread apart. Within his observance of this interference of light, he had established the wave nature of light.

-When Young observed the pattern created on the screen from the light passing through the slits, he didn't find two illuminated regions corresponding to them, but instead saw bright and dark fringes.

-Even though Newton had already proposed that light was a particle, Young's discovery suggested that that wasn't the only thing that could describe the nature of light.



"Double-slit" by Original: NekoJaNekoJa Vector: Johannes Kalliauer is licensed under CC BY- SA 4.0.

How this Experiment makes use of Probability to quantify its outcomes

-Probability comes into play to help predict where these subatomic particles, or the photons in this example, would land in a certain location along the screens and pass through the openings in the screens.

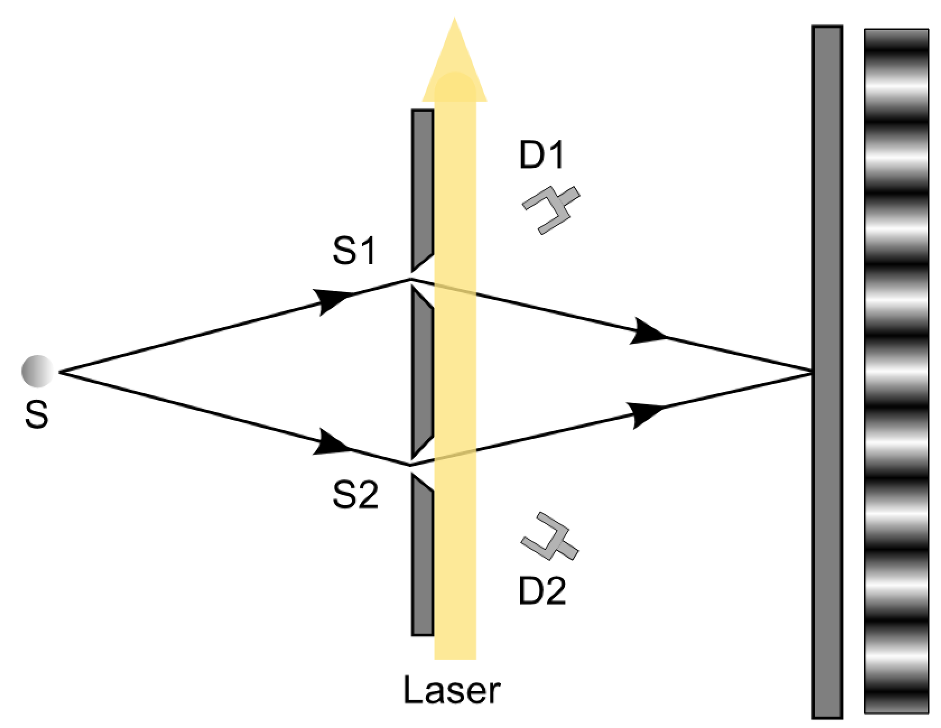
In this experiment, it would also demonstrate where that particle had a higher probability of landing and passing through a certain slit versus where it had a lower probability of landing and passing through a certain slit.

An important thing to take notice of in this experiment is that the scientists who conducted it implied or said that they used a detector, which was a device that measured which slit a particular photon or atom went through before hitting the surface.
The detector is usually placed behind the slits to determine this measurement, labeled as D1 and D2 in the diagram below.

In this experiment, we can label one slit as slit 1(S1) and the other slit on the other side as slit 2 (S2).

-The beam of light is taking two paths in this demonstration, one through slit 1 and the other through slit 2.

-The path of light passing through slit 1 can be labeled as $\Phi1$ while the path of light passing through slit 2 can be labeled as $\Phi2$. $\Phi1$ and $\Phi2$ can also be acknowledged as the wave functions of those two paths of light and describe the probability that the beams of light will pass through the slits.



"File:Double Slit Experiment with detectors.svg" by 老陳 is licensed under CC BY-SA 4.0.

-To put this experiment into more mathematical terms, the total wave function at the detector (ϕdet) can be determined by $\phi1 + \phi2$.

-The probability of detecting where a particle would be located along the screens would come out to this complex equation, $(\phi det)^2 = (\phi1)^2 + (\phi2)^2 + (2|\phi1||\phi2|)(\cos \Delta\xi)$.

-$|\phi1|$ and $|\phi2|$ are the amplitudes of the waves and $\Delta\xi$ is their phase difference at the detector.

-Whether two waves are in phase or not depends on whether they have the same frequency and are aligned with each other. For example, waves in phase with each other have a phase difference of 0.

-When the wavefronts are in phase, $\cos \Delta\xi = 1$ and when they are out of phase, $\cos \Delta\xi = -1$.

-Each of these mathematical concepts would represent a high or low chance of finding the particle.

-In a scenario where one of the slits were closed, slit 1 for example, then $\Phi1$ would be equal to 0 since no path of light would pass through the slit if it were closed.

-There would only be particles present at slit 2 and vice versa if slit 2 was closed. The mathematical equation to represent the closing of slit 1 would be $(\phi det)^2 = (\phi2)^2$ and for slit 2, $(\phi det)^2 = (\phi1)^2$ (Murray 2021).

Experiments #'s 2 and 3

The Wave Equation

-Erwin Schrödinger was an Austrian physicist responsible for the founding of the wave equation, which included the wave function.

-Schrödinger would eventually set up an experiment that demonstrated the diffraction of certain electrons, that would lead to his theory that an electron in an atom would move as a wave, which would be known as the Schrödinger wave equation.

-The Schrödinger wave equation can be expressed as $H\Psi = E\Psi$.

-The Greek letter psi, or the wave function in this equation is known as Ψ.

-The wave function represents the spatial probability of the particle.

-In simpler terms, The wave function, or Ψ, is also known as a probability amplitude which is a mathematical quantity that describes the likelihood of finding a particle, whether it be along a screen similar to the double slit experiment or in any general location.

-The probability of the location of a certain subatomic particle is equal to the square of the wave function in this example, also known as Ψ.

-Tying conditional probability into this example, we can represent the probability amplitude like this: (Particle arrives at some given location | Particle leaves at some given location), representing the probability that the particle arrives at some given location if the location where the particle leaves is known.

The Born Rule

-Max Born was a German physicist who was born the year of 1882 and was homeschooled for the majority of his life.

-Born's time spent studying at university led to the formation of many different theories and ideas, which gave way to his theory that the wave function, based on the Schrödinger equation, should be interpreted as the probability amplitude of finding that particle at a specific location.

-By this, he meant that the wave equation was a mathematical tool for calculating and quantifying the chances of observing a specific outcome in an experiment.

-Born's arguments sum up to the fact that the amplitude of this wave function is related to probability in that it is the probability where you would find the particle at some given location.

-To mathematically represent Born's theory, it focuses on the square of the wave function, $\Psi^2 = \Psi * \Psi$.

-It then goes on to state the absolute value of the wave function squared is proportional to the probability of finding a particle at that point.


"Erwin_Schrödinger" by ditasst is licensed under CC BY-ND 2.0.

Laplace's Demon

-Pierre-Simon Laplace was an astronomer and mathematician, whose work would become incredibly significant, especially quantum mechanics.

-Laplace proposed the idea that if we knew every movement of every subatomic particle in the universe to the utmost precision, we would almost be able to predict the future.

-With the discovery of quantum mechanics, the question of whether the universe behaves probabilistically was challenged as particles of light were noted to behave in a wave-like manner.

-Predictions of those locations may come easier to us with the use of mathematical concepts and probability calculations such as the wave function and Schrödinger's equation and even the mathematics behind Thomas Young's double slit experiment.

Conclusion

-In quantum mechanics, we are supplied with certain tools that can help us pinpoint certain locations of subatomic particles.

-With the help of probability, the mathematical concepts and ideas behind Thomas Young's double slit experiment, the Schrödinger equation, and the Born rule, they are all tools that we can use to help calculate certain measurement outcomes in quantum mechanics.

-Because we are supplied with these concepts and theories, if we reintroduce the ideas from Laplace's demon and the theme of knowing every location of every subatomic particle, then maybe it is possible to predict the future after all.

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