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Mixed Logical and Probabilistic Reasoning in the Game of Clue

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Abstract

Neller and Ziqian Luo ’18 presented a means of mixed logical and probabilistic reasoning with knowledge in the popular deductive mystery game Clue. Using at-least constraints, we more efficiently represented and reasoned about cardinality constraints on Clue card deal knowledge, and then employed a WalkSAT-based solution sampling algorithm with a tabu search metaheuristic in order to estimate the probabilities of unknown card places.

Keywords

Clue, game, reasoning, logic, probability

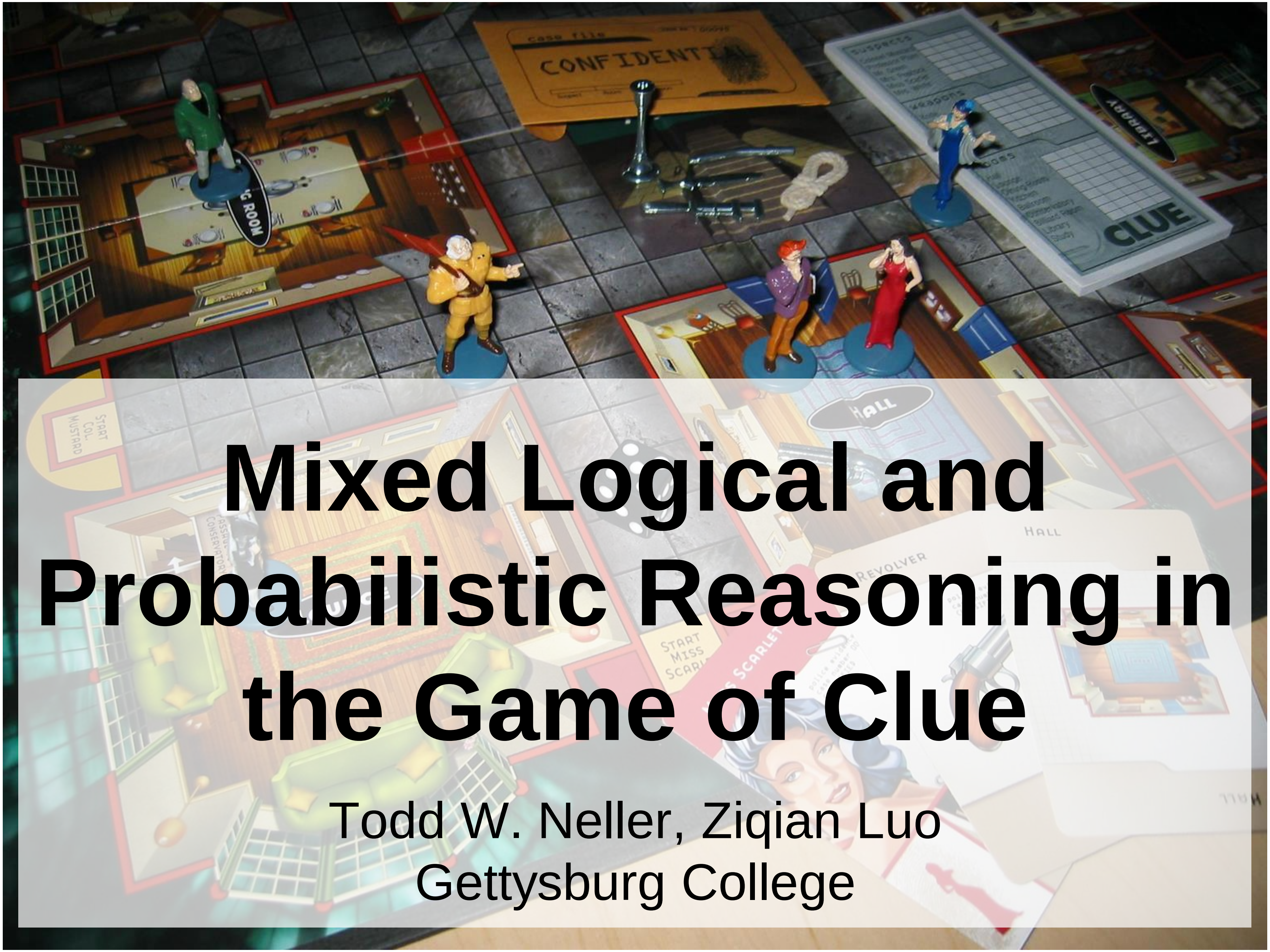
Disciplines

Computer Sciences | Game Design | Probability

Comments

This presentation was given at the 10th International Conference on Computers and Games (CG2018) at National Taipei University in New Taipei City, Taiwan on July 10, 2018.

The authors have also published a journal article based on this presentation.



Mixed Logical and Probabilistic Reasoning in the Game of Clue

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Overview

- Clue/Cluedo Rules
- Basic Propositional Game Knowledge
- At-Least Constraints for Logical Reasoning
- Probabilistic Estimation through WalkSAT-like Sampling
- Algorithmic Variations and Experimental Results
- Conclusions and Future Work

case file
CONFIDENTI
CLUE
LIBRARY
G ROOM
HALL
START COL. MUSTARD
LOUNGE
CONSERVATORY
START MISS SCARL
MISS SCARLET
police evidence case number 00 CLASSIFIED
REVOLVER
HALL

The Game of Clue (a.k.a. Cluedo)

- 21 cards: 6 suspects, 6 weapons, 9 rooms
- Case file has unknown, random suspect, weapon, and room (SWR)
- Remaining cards dealt to players
- Player suggests SWR, first player clockwise that can refute, must show card
- Each player can make 1 SWR accusation
- Correct → win; incorrect → lose (& refute)

Clue Knowledge Representation

- Basic Clue reasoning is constraint satisfaction.
- One formulation: Boolean variables c_p denoting “Card *c* is in place *p*.”
- Given CNF representation of Boolean constraints, one can reason with SAT solver refutations.
- However, not all game knowledge can be expressed in SAT efficiently...

Basic Propositional Game Knowledge

- Initial knowledge
  - Each card is in exactly one place.
  - Exactly one card of each category is in the case file.
  - You know your hand of cards.
  - You know how many cards have been dealt to each player.
- Play knowledge
  - A player cannot refute a suggestion.
  - A player refutes your suggestion by showing you a card.
  - A player refutes another player's suggestion by showing them a card privately.
  - A player makes an accusation and shares whether or not the accusation was correct.

Which of these leads to the largest number of SAT clauses in a CNF representation?

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At-Least Constraints for Logical Reasoning

- Too simple: SAT clauses
  - 6-player game where each player has exactly 3 of the 21 cards.
  - At most 3 cards are held by a player *p*: **35,910 CNF SAT clauses**
  - At least 3 cards are held by a player *p*: **1,260 CNF SAT clauses**
- Too complex/expressive: linear psuedo-Boolean constraints
  - $\sum_i a_i \cdot l_i \ \# \ b$ where a_i, b are integers, l_i are 0/1 literals, and # is an (in)equality.
- Just right: at-least constraints (a.k.a. cardinality constraints, extended clauses)
  - $\sum_i l_i \ \geq b$ where b is an integer and l_i are 0/1 literals.
  - **"At least $\boldsymbol{b}$ of literals $\{\boldsymbol{l_1}, \dots, \boldsymbol{l_n}\}$ are true."**
  - 6-player game where each player has exactly 3 of the 21 cards → only **12 at-least clauses** needed.
  - We have extended Donald Knuth's Dancing Links (DLX) algorithm for more general constraint satisfaction with at-least constraints.

Probabilistic Estimation through Sampling

- Probabilities for unknown card positions can be *exactly computed* with model counting.
  - Model counting is combinatorially infeasible for all but endgame scenarios with few models.
- Probabilities for unknown card positions can be *approximately computed* with model sampling.
- *WalkSAT step*:
  - Pick a random unsatisfied constraint clause.
    - Flip a variable chosen at random from among those that would cause the fewest clauses to become unsatisfied.
- *Tabu metaheuristic*: A tabu tenure is the number of steps that must pass before a variable may be flipped again.

Algorithm 1 Search and Sample — a WalkSAT-like sampling algorithm

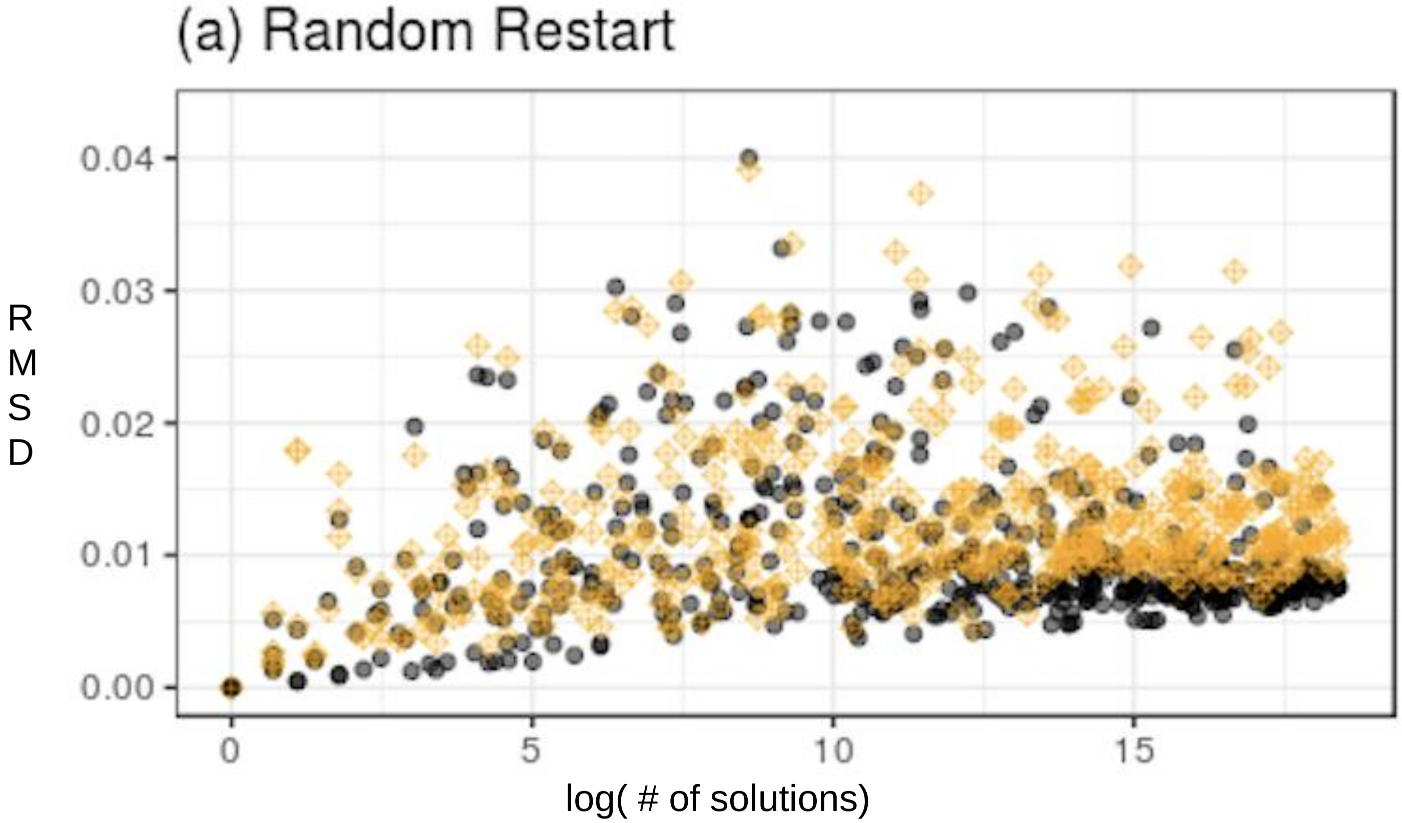
```
Input: at-least clauses of the game state, fixed variable assignments, number of search iterations
Output: probability estimate of each card being at each place

function SearchAndSample($clauses, fixedVarAssignments, numIter$):
RandomRestart(); {randomly assign all non-fixed variables}
if there are no unsatisfied clauses then
    RecordSample(); {tally true non-fixed variables, increment sample count}
end if
$tabuTenure = 10$
$lastSampleStep = 0$
for $step = \{1, 2, 3, \ldots, numIter\}$ do
    $tabuCutoffStep = \max(1, step - tabuTenure, lastSampleStep)$
    if all clauses are satisfied then
        Choose a random non-fixed variable.
    else
        Choose a random unsatisfied clause.
        Choose a variable from that clause according to the WalkSAT heuristic.
    end if
    Flip(); {make chosen variable flip assignment, update data structures}
    if there are no unsatisfied clauses then
        RecordSample();
        $lastSampleStep = step$
    end if
```

Algorithmic Variations

- Our testing revealed two problems that cause probabilistic approximation bias:
  - In opening game states:
    - Too high a tabu tenure results in too few samples.
    - Too low a tabu tenure results in too many returns to the same sample, and too few unique samples.
  - In endgame states:
    - Even when all solutions can be sampled, WalkSAT-like sampling is still non-uniform and biased.

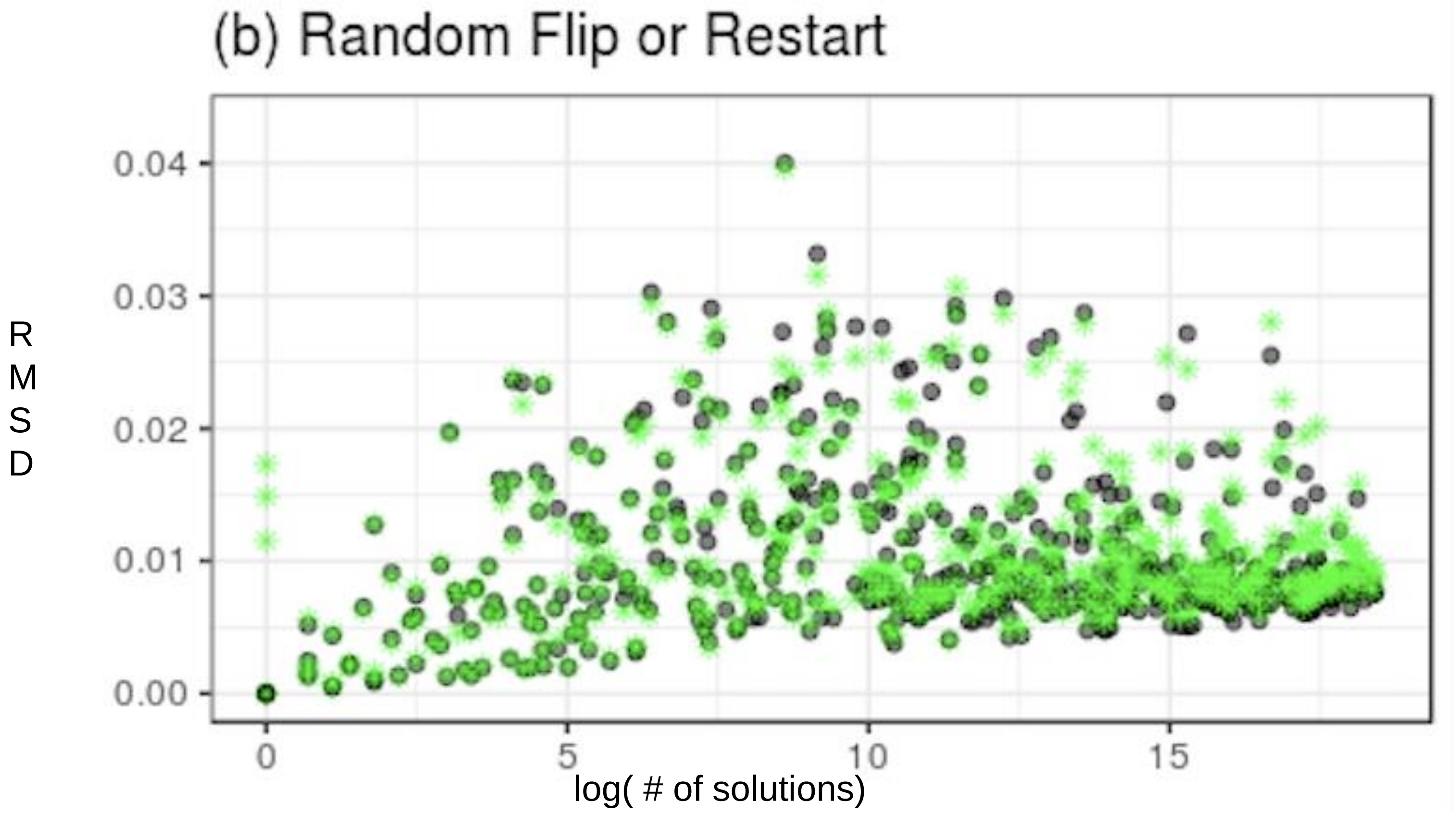
Algorithmic Variations

Random Restart: After finding and recording a sample solution, perform a random restart, reinitializing all non-fixed variable to random values.



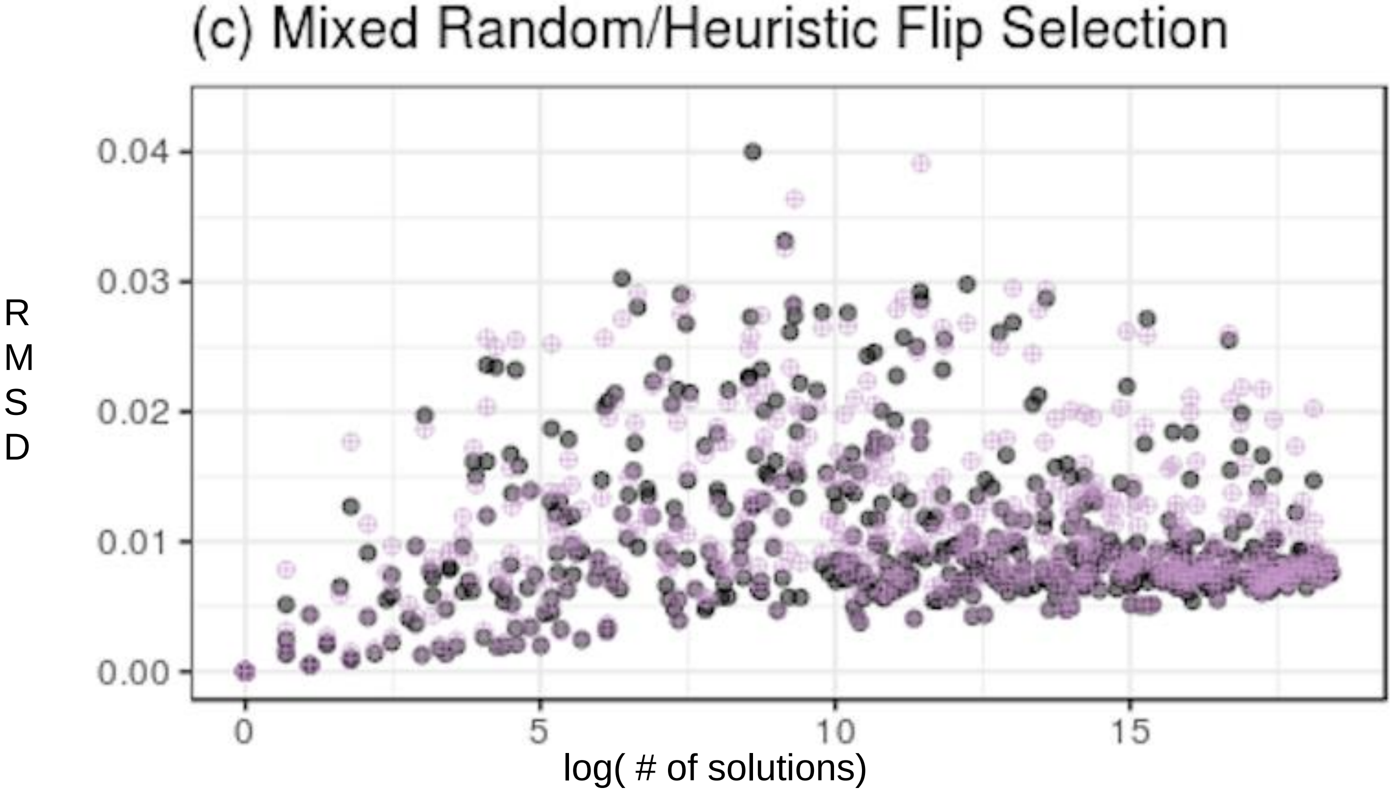
Algorithmic Variations

Random Flip or Restart: After finding and recording a sample solution, perform a random variable flip with probability 0.2. Otherwise, perform a random restart.



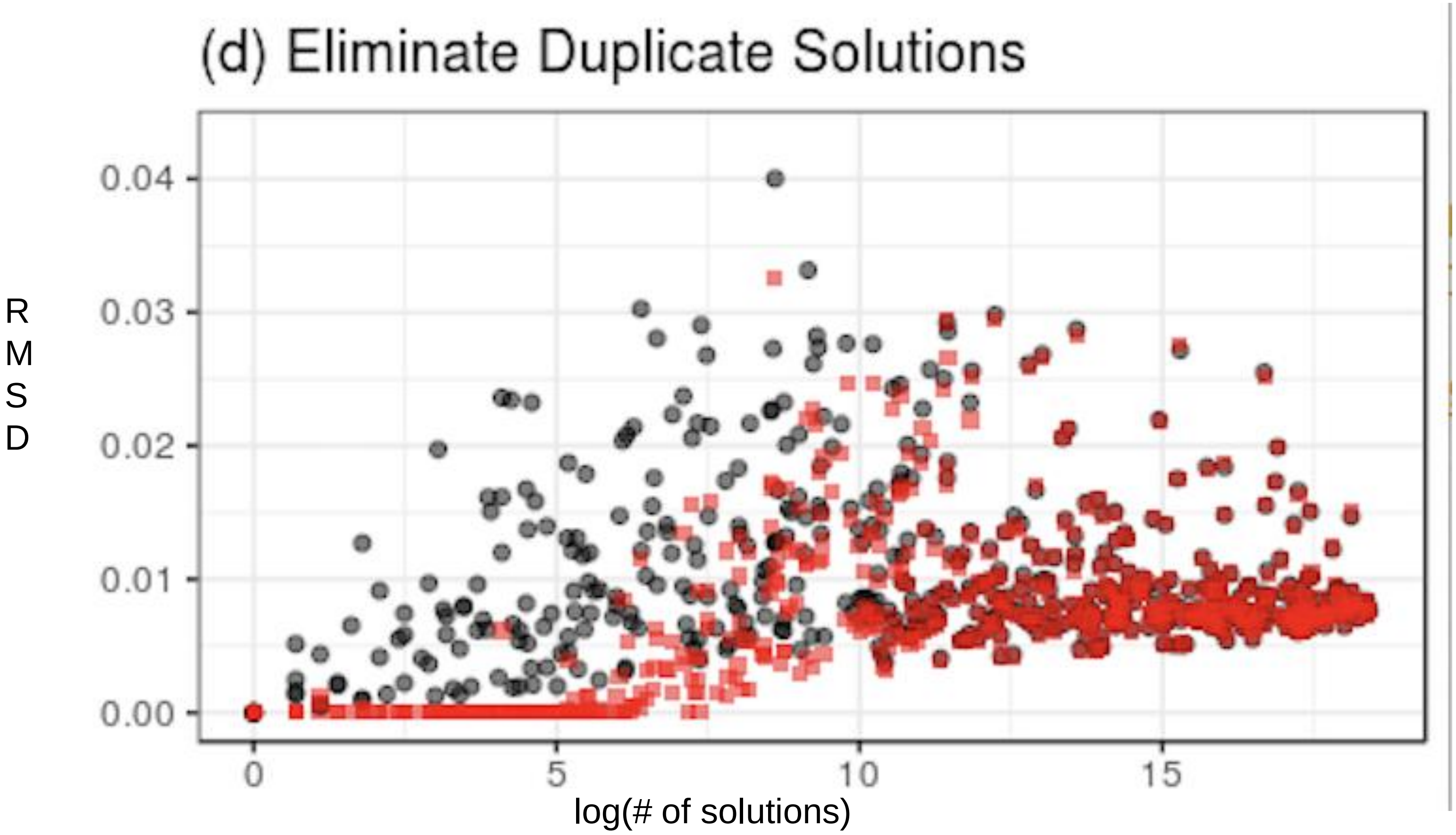
Algorithmic Variations

Mixed Random/Heuristic Flip Selection: After having chosen a random unsatisfied clause, with probability 0.2, flip a random variable of that clause. Otherwise, flip a random variable among those that minimize the number of clauses that will become false as a result.



Algorithmic Variations

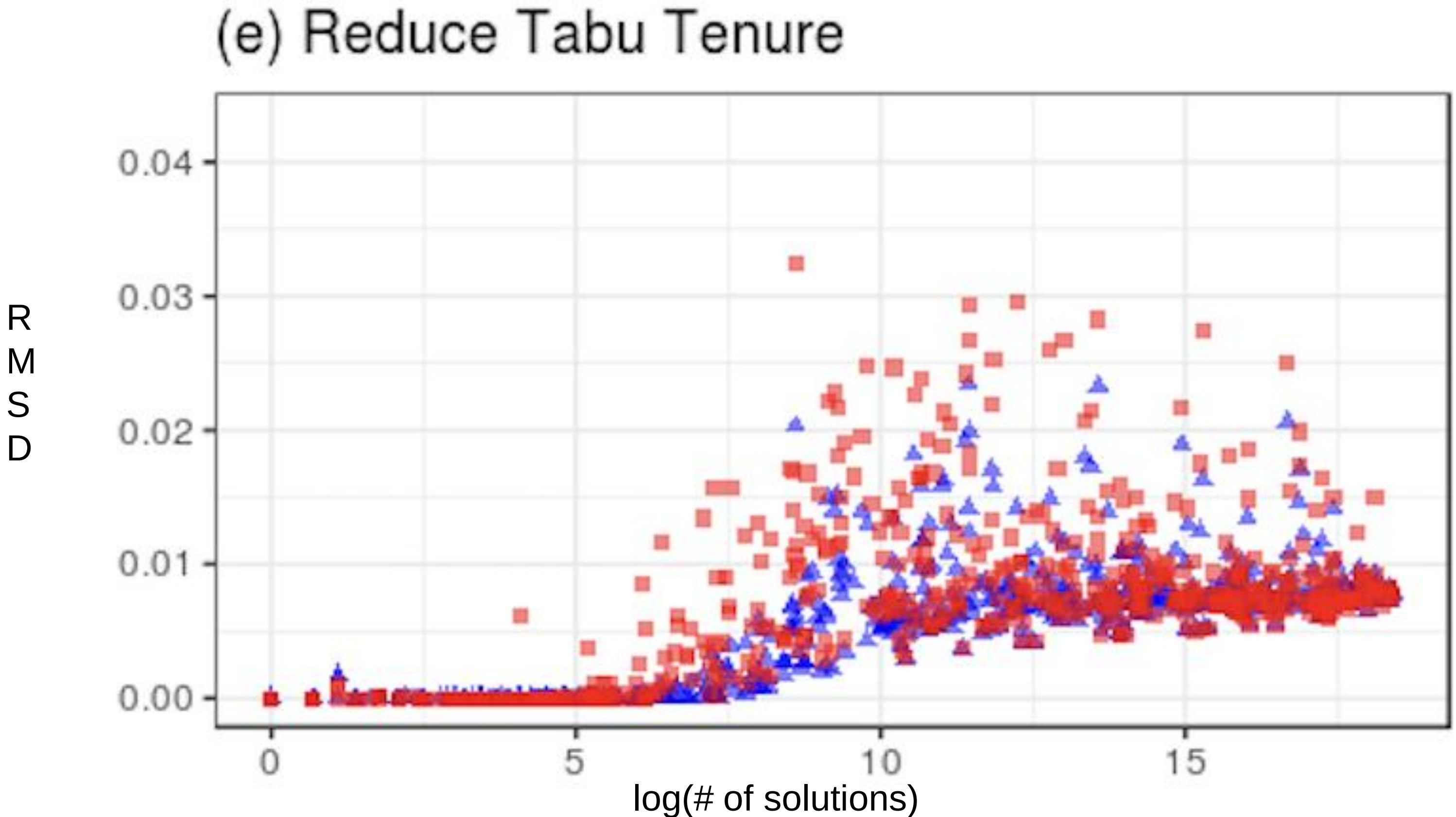
Eliminate Duplicate Solutions: Record only unique sample solutions.



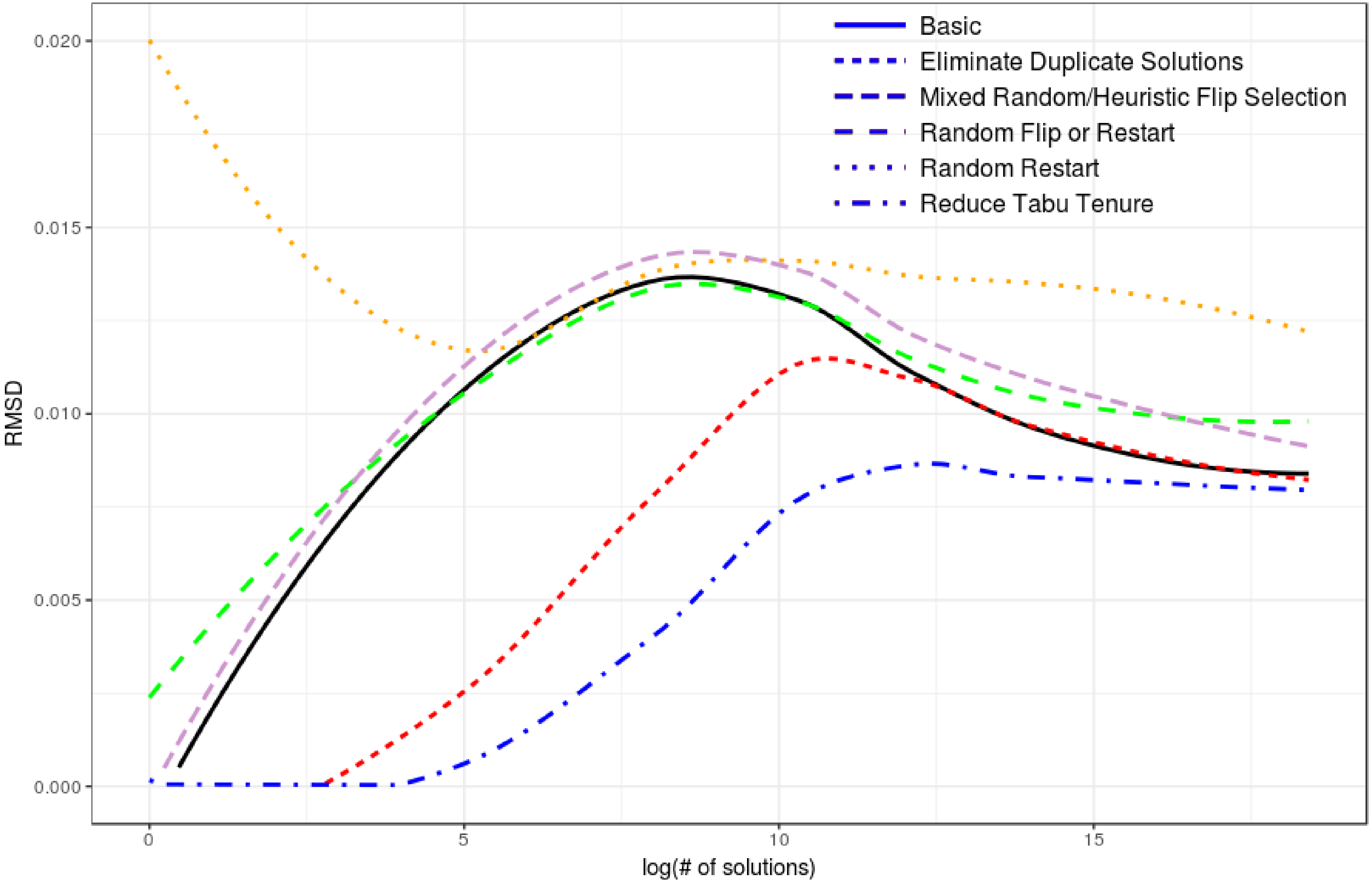
(d) Eliminate Duplicate Solutions

Algorithmic Variations

Reduce Tabu Tenure: In addition to unique samples, reduce the tabuTenure constant from 10 to 2, allowing greater frequency of individual variable flips.



Experimental Results



Conclusions

- The efficiency of finding and sampling solutions with a WalkSAT-like heuristic is also the cause of sampling bias.
- Two ideas resulted in a total 41% reduction of root-mean-square deviation in estimation error:
    - elimination of duplicate samples
    - reduction in tabu tenure - the tabu metaheuristic was important, yet the best tabu tenure was a short tenure for this problem domain.
- Seeking a more diverse sample through the introduction of various forms of randomness came at an even greater cost of error through much-reduced sampling.

Future Work

- This work represents initial steps to mitigate such sampling bias and compute better probabilistic estimates efficiently.
- However, we would expect that future work could improve upon this work in two important respects described in (Gomes, 2009):
  - estimation quality (i.e. through improvements such as we’ve found), and
  - confidence bounds on such estimations.
- Such confidence bounds are of interest in assessing the utility of making, for example, an uncertain accusation when one believes one may not get another turn to make a certain accusation in Clue.

Questions?