

2023

## The Effect of Remote Work on Firm Level Productivity

Katherine Fullowan  
*Gettysburg College*

Follow this and additional works at: <https://cupola.gettysburg.edu/ger>



Part of the [Economic Theory Commons](#), and the [Labor Economics Commons](#)

**Share feedback** about the accessibility of this item.

---

### Recommended Citation

Fullowan, Katherine (2023) "The Effect of Remote Work on Firm Level Productivity," *Gettysburg Economic Review*: Vol. 12, Article 4.

Available at: <https://cupola.gettysburg.edu/ger/vol12/iss1/4>

This open access article is brought to you by The Cupola: Scholarship at Gettysburg College. It has been accepted for inclusion by an authorized administrator of The Cupola. For more information, please contact [cupola@gettysburg.edu](mailto:cupola@gettysburg.edu).

---

## The Effect of Remote Work on Firm Level Productivity

### Abstract

This paper investigates the impact of remote work on firm-level productivity. To observe this trend, we develop a theoretical model to understand how an economy performs. We consider the economy as a collection of firms in an attempt to maximize profit. By observing a firm's profit function, we are able to derive their productivity by maximizing a representative firm's profit function. For simplicity purposes, this study treats labor as the only factor of production to focus solely on how changes in the number of remote workers impact productivity. We ultimately find that productivity increases when the number of remote workers increases relative to non-remote workers. This holds true under the stipulation that remote workers are paid higher wages than non-remote workers.

### Keywords

work from home, productivity, theoretical analysis, efficiency wage theory

# The Effect of Remote Work on Firm Level Productivity

Katie Fullowan

May 1, 2023

## Abstract

This paper investigates the impact of remote work on firm-level productivity. To observe this trend, we develop a theoretical model to understand how an economy performs. We consider the economy as a collection of firms in an attempt to maximize profit. By observing a firm's profit function, we are able to derive their productivity by maximizing a representative firm's profit function. For simplicity purposes, this study treats labor as the only factor of production to focus solely on how changes in the number of remote workers impact productivity. We ultimately find that productivity increases when the number of remote workers increases relative to non-remote workers. This holds true under the stipulation that remote workers are paid higher wages than non-remote workers.

**Keywords** Work from Home · Productivity · Theoretical Analysis · Efficiency Wage Theory

**Acknowledgements:** I would like to thank Professor Hu for her feedback and assistance with this research paper.

*I affirm that I will uphold the highest principles of honesty and integrity in all my endeavors at Gettysburg College and foster an atmosphere of mutual respect within and beyond the classroom.*

## 1. Introduction

A recent shift towards remote work has made it increasingly more important for researchers to understand how the economy is being affected. This paper observes the impact of remote work on productivity with theoretical analysis. Furthermore, we look at how wages paid to remote and non-remote workers influence a firm's productivity. The growing presence of remote work around the world was accelerated in March of 2020 when the Coronavirus disease 19 (Covid-19) struck the United States with force.

The U.S. reported its first confirmed case of Covid-19 on January 20, 2020, with the first reported death occurring about a month later. The positive case count exceeded to a total of 60 cases across 12 different states by March 3<sup>rd</sup>. Between March 11<sup>th</sup> to March 19<sup>th</sup>, the World Health Organization declared Covid-19 a worldwide pandemic, the U.S. declared a nationwide emergency, public school systems began shutting down, and states began issuing mandatory stay-at-home orders. By April 10<sup>th</sup>, 2020, over 500,000 cases were reported in the U.S. alone, with the death count exceeding 18,600 (Center for Disease Control and Prevention, 2022).

Not only did the Covid-19 pandemic have significant immediate health effects, but it also seriously impacted employment. From mid-March to the end of April, over 26.5 million people in the U.S. became unemployed. By May 9<sup>th</sup>, unemployment rates reached their highest levels since the Great Depression at

14.7% and roughly 20.5 million workers from the Arts, Entertainment, and Recreation industry were out of work (Center for Disease Control and Prevention, 2022). Between issued stay-at-home orders and a spike in unemployment rates, not only were many people looking to minimize spending, but there was also little to spend money on. Travel was heavily restricted, many non-essential businesses were temporarily shut down, and people overall were scared to go out in public and risk exposure.

This major demand shock was brought on by the combination of stay-at-home orders and Covid cases. Stay-at-home orders forced many individuals to make a shift to remote work where possible. In 2021, about 17.9% of Americans were primarily working from home. This number tripled since 2019 when roughly only 5.7% of Americans primarily worked from home. The percentage of individuals working from home varied greatly by region with upwards of 48.3% of workers in the District of Columbia working remotely (US Census Bureau, 2022). Thus, remote work not only has grown in response to the Covid-19 pandemic, but it is still heavily prevalent across the United States. It is essential to understand if this recent movement towards remote work has an impact on productivity.

This paper is unique from most other in the way it develops a theoretical model to study how an economy performs. More specifically, it attempts to understand the impact of remote work on productivity. To do so, we consider the

economy as a collection of firms with an attempt to maximize its profit function, shown in Equation 3. Maximizing profit allows us to find out the optimal number of remote and non-remote workers. This value can then be substituted into the model for output, shown in Equations 5 and 6. By dividing this output model by workers in the labor force, I generate a model for productivity which is further analyzed to answer the research question. In doing so, this study ultimately finds that when the ratio of remote to non-remote workers increases, firm productivity is positively impacted. Conversely, if this ratio decreases, meaning the number of remote workers is declining with respect to non-remote workers, then productivity is negatively impacted.

Section 2 of this paper dives into relevant literature, followed by the development of the theoretical model in Section 3. Results and a discussion of results from the modeling section are covered in Section 4, with concluding remarks included in Section 5.

## **2. Literature Review**

The urgent need to adjust to working throughout a pandemic ultimately shifted how people worked, whether it be at limited capacity in the grocery store or at home with a house full of children. Workers adapted and found ways to work under the new circumstances. In some studies, work from home (WFH) has been found to improve work performance, in addition to increasing job

satisfaction (Bloom et al., 2022). In this study by Bloom et al. researchers use data on employee's six-month performance review in conjunction with promotion rates for engineers and finance and marketing employees at a technology firm. Bloom et al. found that when employees worked from home a couple days per week, they reported 33% less attrition and higher levels of job satisfaction. Researchers subsequently found that non-managers were not only more likely to volunteer for remote work, but also to report experiencing positive productivity impacts. Managers on the other hand, were found to be less likely to volunteer for remote work and to be more likely to quit their job when asked to work remotely. Since managers are responsible for the oversight of their employees, it appears reasonable to conclude that they would prefer working in closer proximity to their workers.

A study by Morikawa (2020) found contradictory evidence that average productivity from home was lower than that in the office. In this study, Morikawa used data from a survey in June of 2020 on prevalence, frequency, and productivity of work from home. Morikawa ultimately finds that the average productivity when working from home was roughly 60-70% of normal, in-office productivity levels. Furthermore, he finds that productivity was even lower for workers that only started working remotely after the pandemic had begun.

Another by Felstead and Rueschke (2020) found that there was little impact at all of WFH on productivity. When conducting their research, Felstead and

Rueschke look at data from a report that observes at-home work before and during the lockdown in the UK. Results from a survey described in the data set includes individuals reported levels of productivity in comparison to before they made the shift to remote work. This survey found conflicting evidence with 40.9% reporting getting as much work done at home, 28.9% reporting getting more done, and 30.2% reporting getting less done. These conflicting findings are likely explained by the wide range of occupations included.

A study performed by Kitagawa et al. (2021) uses empirical analysis to observe whether productivity changed for workers that had to WFH because of the Pandemic. They observe changes in productivity levels, similar to what is done in this study. Some key differences between this paper and mine are its empirical nature and the unit of study being individual vs. firm. The study used self-reported data from a manufacturing company in Japan which may largely contribute to the results. Kitagawa et al. find that workers working from home reported declines in productivity, largely due to poor office set up and internet connection. This data was collected in April and June of 2020, likely before workers had chances to upgrade their office set up at home.

Many papers, such as those by Bloom et al. (2020), Morikawa (2020), Felstead and Rueschke (2020), and Kitagawa (2021), that look at working from home and productivity levels take an empirical approach. A study by Zhang et al. (2021) takes both an empirical and theoretical approach to understanding when



firm choose to WFH, setting it apart from many other studies. In their empirical analysis, researchers use a data set that follows small businesses and their performance. In this analysis, Zhang et al. find that WFH rates increased even after stay-at-home orders were no longer in place. Therefore, even after workers were permitted to return to the office, working preferences shifted. This further emphasizes the importance of understanding any changes in productivity that may result. More relevant to this study, they also find that rational employers would select to WFH as opposed to work in the office. Furthermore, in states with higher WFH rates, small businesses performed better overall (after controlling for various factors).

While this paper also takes a theoretical approach, the two are done very differently and reach different conclusions. In Zhang et al.'s theoretical analysis, they predict that firms would choose to allow WFH if the ratio of variable revenue to cost is greater in WFH setting than in a standard office setting. Thus, their model uses a firm-revenue expense accounting framework, considering four key factors of production (labor, capital, land, and entrepreneurship). In this paper I simplify my analysis by looking at just one factor of production, labor. My major finding is consistent with the wage efficiency theory which indicates that in hiring more remote workers, firms tend to improve their productivity. This largely occurs because remote workers become more productive when paid better wages than non-remote workers.

### 3. Modeling

To study how remote working affects productivity, I consider an economy that consists of a collection of firms. Each firm is assumed to be homogenous with an attempt to maximize its profit. For simplicity, a representative firm is assumed to produce by only relying on one production factor – labor. Labor is further divided up into two types, including labor provided by remote workers and non-remote workers, as noted in Equations 1 and 7. The firm then adopts technology to combine these two types of labor to produce.

As stated by basic economic theory, a firm's profit is defined as the difference between total revenue and total cost. Thus, to formulate a profit function, I look at how much total revenue and total cost the representative firm earns and incurs, respectively.

Total revenue is defined in economics as the product of price and quantity. Equation 1 below represents total revenue as a function of average price level ( $P_t$ ) and quantity produced, or output ( $Y_t$ ). Output in this model is measured by a Cobb-Douglass production function with inputs ( $L_{1t}$  and  $L_{2t}$ ) and technology ( $A_t$ ), defined in Equation 5 below.  $\beta$  and  $\delta$  are elasticities of output while  $L_{1t}$  and  $L_{2t}$  are the number of remote and non-remote workers, respectively. For simplicity purposes, the model used in this paper does not consider capital as a factor of production. Rather, the model focuses solely on the impact of changes in number of remote and non-remote workers on profit and, ultimately, on

productivity. The exponents of  $\beta$  and  $\delta$  represent elasticity of output with respect to each group of workers.

$$TR = P_t A_t L_{1t}^\beta L_{2t}^\delta \quad (1)$$

where  $L_{1t}$  = number of remote workers

$L_{2t}$  = number of non – remote workers

$P_t$  = average price of all goods and services produced

$A_t$  = total factor productivity

As previously indicated, the models observed in this paper only consider labor as a factor of production. Thus, total costs is a function of workers’ wages and number of workers. The products of W and T below show the product of number of remote and non-remote workers and their respective wages. The sum of these products gives us our total cost equation below.

$$TC = W_1 L_{1t} + W_2 L_{2t} \quad (2)$$

where  $W_1$  = wages paid to remote workers

$W_2$  = wages paid to non – remote workers

Putting together the equations for total revenue and total cost, we reach Equation 3 below, which models the nation’s profit. The constraint on equation 3 denotes  $\alpha$  as the ratio between the number of remote and non-remote workers. Ultimately this constraint indicates that as the workforce transitions between remote and non-remote work, profits vary as well.

$$\max \Pi_t = P_t A_t L_{1t}^\beta L_{2t}^\delta - W_1 L_{1t} - W_2 L_{2t} \quad (3)$$

$$\text{s.t. } \frac{L_{1t}}{L_{2t}} = \alpha \rightarrow L_{1t} = \alpha L_{2t}$$

After substituting  $L_{1t}$  into the above profit function, using first order conditions with respect to  $L_{2t}$ , and solving for  $L_{2t}$ , we reach the following. See appendix for more detailed steps.

$$L_{2t}^* = \left( \frac{\alpha W_1 + W_2}{(\beta + \delta) P_t A_t \alpha^\beta} \right)^{\frac{1}{\beta + \delta - 1}} \quad (4)$$

As previously noted, output ( $Y_t$ ) is modeled in a Cobb-Douglas production function. Keeping in mind that  $L_{1t} = \alpha L_{2t}$ ,  $L_{2t}^*$  can be substituted into the equation for output. Upon doing so, we derive the following equations.

$$Y_t^* = A_t L_{1t}^\beta L_{2t}^\delta \rightarrow Y_t^* = A_t \alpha^\beta L_{2t}^{\beta + \delta} \quad (5)$$

$$Y_t^* = A_t \alpha^\beta \left( \frac{\alpha W_1 + W_2}{(\beta + \delta) P_t A_t \alpha^\beta} \right)^{\frac{1}{\beta + \delta - 1} (\beta + \delta)} \quad (6)$$

The Bureau of Labor Statistics defines productivity as output divided by input. Therefore, we can calculate productivity as demonstrated in Equation 7 below. This is further simplified in equation 8. As previously mentioned, this paper does not consider capital as a factor of production for both the sake of the research question and simplicity. Thus,  $L_t$  represents total input or total number of workers in the economy and thus is the sum  $L_{1t}$  and  $L_{2t}$ .

$$y_t^* = \frac{Y_t}{L_t}, \text{ where } L_t = L_{1t} + L_{2t} \quad (7)$$

After dividing  $Y_t$  by  $L_t$  and simplifying, Equation 8 is reached. See appendix for more detailed steps.

$$y_t^* = \frac{\left(\frac{\alpha W_1 + W_2}{(\beta + \delta) P_t}\right)}{(\alpha + 1)} \rightarrow y_t^* = \frac{\alpha W_1 + W_2}{(\beta + \delta)(\alpha + 1) P_t} \quad (8)$$

Since this productivity,  $y_t^*$ , equation is a function of  $\alpha$ , we can take the partial derivative with respect to  $\alpha$  to determine how productivity varies with changes in the ratio of the number of remote to non-remote workers. We use comparative statistical analysis, holding exogenous variables constant and allowing  $\alpha$  to vary. In doing so, we can understand how a changing  $\alpha$  changes with productivity,  $y_t^*$ . After performing the derivation, Equation 9 is reached. See appendix for further steps.

$$\frac{dy_t^*}{d\alpha} = \frac{1}{(\beta + \delta) P_t (\alpha + 1)^2} (W_1 - W_2) \quad (9)$$

This equation demonstrates the relationship between  $\alpha$  and productivity,  $y_t^*$ . By observing this relationship between productivity and the ratio between remote and non-remote workers, we are able to understand how a changing ratio impacts productivity. This model will be further interpreted in section 1.4 below.

#### 4. Results and Discussion

As can be seen in Equation 9,  $\frac{dy_t^*}{d\alpha}$  takes on positive values for all  $W_1 > W_2$ . This is determined by analyzing the values of each component of the function. The values of  $\beta$  and  $\delta$  represent output elasticities and thus, are assumed

to be positive. These values are summed and multiplied by  $P_t$ , which represents price level which must be positive. These values are further multiplied by the sum of  $\alpha$  and 1 squared. Since  $\alpha$  is the ratio between two populations of workers, it too must be positive. While this product is in the denominator, the value does not change. Therefore, since  $\frac{1}{(\beta+\delta)P_t(\alpha+1)^2} > 0$ , the value of  $\frac{dy_t^*}{d\alpha}$  depends solely on  $W_1$  and  $W_2$ .

So, if the wages paid to remote workers is higher than the wages paid to non-remote workers, then changes in productivity will be positive. This positive value indicates that as the number of remote workers increases, relative to non-remote workers, productivity will also increase. Conversely, if the number of remote workers decrease, relative to non-remote workers, productivity will also decrease. This positive relation between productivity and remote work falls in line with the findings from Bloom et al.'s empirical analysis (2022).

These results are consistent with the efficiency wage theory. Essentially, this theory states that firms are willing to pay individuals higher wages to retain workers and will make them less likely not to work (Shapiro and Stiglitz, 1984). If workers are paid more, then they are motivated to work harder to maintain their jobs. Thus, as proven in this paper, higher wages paid to workers are positively associated with productivity. The theory further acknowledges that with higher wages paid to workers, there is less of a need to closely monitor workers (Shapiro

and Stiglitz, 1984). When employees work remotely, there is inherently less supervision. By paying remote workers higher wages, firms can ensure that their employees are working harder than they would if they were paid less.

These findings only hold true if  $W_1$  is in fact greater than  $W_2$ . Since the onset of the pandemic, highly educated and high-income workers were more likely to maintain their job and to work remotely (Bick et al., 2020 & Dingel and Neiman, 2020). Thus, it is likely those remote workers categorized by  $L_{1t}$  earn wages higher than non-remote workers categorized by  $L_{2t}$ . In other words, it is likely that  $W_1 > W_2$ .

If  $W_1 < W_2$  then the value of  $\frac{dy_t^*}{d\alpha}$  takes on negative values. This means that if non-remote workers are paid higher wages than remote workers, productivity would decrease. If remote workers are typically more highly educated than non-remote workers, then they may become discouraged from working their lower paying jobs. Rather, they would be incentivized to leave their current role in remote work and take an in-person job to be paid higher wages, all else equal. In doing so, the productivity of the firm would be negatively impacted.

## 5. Conclusion

In this study I find that hiring more remote workers will lead to higher firm productivity. Conversely, the study also finds that hiring more remote workers will lead productivity to decline instead. However, a study by Bick et al.

(2020) found that higher paid workers are more likely to work remotely. Thus, it is plausible to conclude that it is likely for remote wages to be higher than non-remote wages.

This paper contributes to prior literature in its theoretical nature. Previous studies, such as those by Felstead and Rueschke (2020), Kitagawa et al. (2021), Morikawa (2020), and Hipp and Bünning (2020), all take an empirical approach to observing productivity levels during the pandemic. A study by Zhang et al. (2020) used both a theoretical and empirical approach to observe when firms select to WFH. While they used theory, the results focused on whether employers choose WFH or work in an office setting as opposed to how differing wages influence productivity.

The findings of this study have important implications for firms. We prove that if firms pay wages to remote workers that are higher than those paid to non-remote workers, then productivity will increase. Thus, if firms are aware of these results, they can offer higher wages to employees to work from home. In doing so, they incentivize workers to work harder while unsupervised to maintain their job. This is consistent with the efficiency wage theory that paying higher wages increases the opportunity cost of not working (Bowles, 1981 and Eaton and White, 1982).



## 6. References

- Bick, A., Blandin, A., & Mertens, K. (2020). Work from Home After the COVID-19 Outbreak. *Federal Reserve Bank of Dallas, Working Papers, 2020*(2017). <https://doi.org/10.24149/wp2017r1>
- Bloom, N., Han, R., & Liang, J. (2022, July). How hybrid working from Home Works Out. *NBER Working Paper Series*, (30292). doi:10.3386/w30292
- Bowles, S (1981). Technical change and the profit rate: A simple proof of the Okishio theorem. *Cambridge Journal of Economics*, 5(2), 183–186.
- Bullard, J. (2021, December 09). Long-run GDP growth requires higher productivity growth: St. Louis Fed. Retrieved December 7, 2022, from <https://www.stlouisfed.org/publications/regional-economist/october-2016/higher-gdp-growth-in-the-long-run-requires-higher-productivity-growth>
- Center for Disease Control and Prevention. (2022, August 16). CDC Museum Covid-19 Timeline. Retrieved December 8, 2022, from <https://www.cdc.gov/museum/timeline/covid19.html>
- Dingel, J. I., & Neiman, B. (2020). How many jobs can be done at home?. *Journal of Public Economics*, 189, 104235.
- Eaton, CB and W White (1982). Agent compensation and the limits of bonding. *Economic Inquiry*, 20(3), 330–343.
- Felstead Alan & Reuschke Darja. (2020, August). Homeworking in the UK: before and during the 2020 lockdown. WISERD Report, Cardiff: Wales Institute of Social and Economic Research, <https://wiserd.ac.uk/publications/homeworking-uk-and-during-2020-lockdown>.
- Hipp, Lena, and Mareike Bünning. “Parenthood as a Driver of Increased Gender Inequality during Covid-19? Exploratory Evidence from Germany.” *European Societies* 23, no. sup1 (2020). <https://doi.org/10.1080/14616696.2020.1833229>.

- Kitagawa, Ritsu, et al. “Working from Home and Productivity under the COVID-19 Pandemic: Using Survey Data of Four Manufacturing Firms.” *PLOS ONE*, vol. 16, no. 12, 2021, <https://doi.org/10.1371/journal.pone.0261761>.
- Mongey, Simon, et al. (2021). Which Workers Bear the Burden of Social Distancing? *The Journal of Economic Inequality*, vol. 19, no. 3, pp. 509–526., <https://doi.org/10.1007/s10888-021-09487-6>.
- Morikawa, M. (2020). Productivity of Working from Home during the COVID-19 Pandemic: Evidence from an Employee Survey. *RIETI Discussion Paper*, 20(E).
- Shapiro C and JE Stiglitz (1984) Equilibrium unemployment as a worker discipline device. *The American Economic Review*, 74(3), 433–444.
- U.S. Census Bureau. (2022, September 15). The Number of People Primarily Working From Home Tripled Between 2019 and 2021. Retrieved April 16, 2023, from Census.gov website: <https://www.census.gov/newsroom/press-releases/2022/people-working-from-home.html>.
- U.S. Bureau of Labor Statistics. (2020, June 24). What is Productivity? : U.S. Bureau of Labor Statistics. Retrieved from Bls.gov website: <https://www.bls.gov/k12/productivity-101/content/what-is-productivity/home.htm#:~:text=Productivity%20is%20a%20measure%20of>
- Zhang, T., Gerlowski, D., & Acs, Z. (2021). Working from home: small business performance and the COVID-19 pandemic. *Small Business Economics*, 58(2). <https://doi.org/10.1007/s11187-021-00493-6>

## 7. Appendix

The below equation is reached after substituting  $L_{2t}$  into Equation 3 in section 1.3.

$$\max \Pi_t = P_t A_t \alpha^\beta L_{2t}^{\beta+\delta} - L_{2t} (\alpha W_1 + W_2)$$

From here, the first order condition is derived with respect to  $L_{2t}$  below.

This equation is then used in section 1.3 to solve for  $L_{2t}^*$ .

$$\frac{d\Pi_t}{dL_{2t}} = (\beta + \delta) P_t A_t \alpha^\beta L_{2t}^{\beta+\delta-1} - (\alpha W_1 + W_2) = 0$$

Below shows the steps to get from Equation 7 to 8. Here,  $Y_t$  is divided by  $L_t$  which is rewritten as a function of  $L_{2t}$  and  $\alpha$ .

$$L_t = \alpha L_{2t} + L_{2t}$$

$$L_t = L_{2t} (\alpha + 1)$$

$$L_t = \left( \frac{\alpha W_1 + W_2}{(\beta + \delta) P_t A_t \alpha^\beta} \right)^{\frac{1}{\beta + \delta - 1}} (\alpha + 1)$$

$$y_t^* = \frac{A_t \alpha^\beta \left( \frac{\alpha W_1 + W_2}{(\beta + \delta) P_t A_t \alpha^\beta} \right)^{\frac{1}{\beta + \delta - 1} \beta + \delta}}{\left( \frac{\alpha W_1 + W_2}{(\beta + \delta) P_t A_t \alpha^\beta} \right)^{\frac{1}{\beta + \delta - 1}} (\alpha + 1)}$$

$$y_t^* = \frac{A_t \alpha^\beta \left( \frac{\alpha W_1 + W_2}{(\beta + \delta) P_t A_t \alpha^\beta} \right)^{\frac{1}{\beta + \delta - 1} \beta + \delta - 1}}{(\alpha + 1)}$$

$$y_t^* = \frac{A_t \alpha^\beta \left( \frac{\alpha W_1 + W_2}{(\beta + \delta) P_t A_t \alpha^\beta} \right)}{(\alpha + 1)}$$

Below we walk through the steps to reach Equation 9. To do so, the derivative of the productivity equation (8) is taken with respect to  $\alpha$  to show how a changing ratio of remote to non-remote workers will impact productivity.

$$\frac{dy_t^*}{d\alpha} = \frac{1}{\alpha+1} \left( \frac{W_1}{(\beta+\delta)P_t} \right) + \frac{\alpha W_1 + W_2}{(\beta+\delta)P_t} \left( \frac{-1}{(\alpha+1)^2} \right)$$

$$\frac{dy_t^*}{d\alpha} = \frac{W_1}{(\beta+\delta)P_t} \left( \frac{-\alpha}{(\alpha+1)^2} + \frac{1}{\alpha+1} \right) - \frac{W_2}{(\beta+\delta)P_t(\alpha+1)^2}$$

$$\frac{dy_t^*}{d\alpha} = \frac{W_1}{(\beta+\delta)P_t} \left( \frac{1}{(\alpha+1)^2} \right) - \frac{W_2}{(\beta+\delta)P_t(\alpha+1)^2}$$