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Driven "Portulum": A Rolling Ball as a Simple Oscillating System

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Abstract
A classroom demonstration, a variation of the simple swinging pendulum, is described. In our "portulum," a ball, driven by short blasts of air, rolls along a curved tube. The design of this device, its construction, and its usefulness to the teaching of physics are discussed. It is also shown that the oscillations of the rolling ball have the same mathematical form as the oscillations of the ball swinging along the same path, but with a lower frequency.

Keywords
portulum, simple oscillating system, elementary mechanics

Disciplines
Physics

Stockton has an open laboratory system, which works well for this type of project. An additional commitment to provide laboratory access would be placed on the instructor in those institutions with other laboratory policies.

There is always a problem with unmotivated students in any course. It was a pleasure to see students who were normally unexcited by more traditional classroom techniques perform well on this project. Some students were motivated by the "fun" aspect of the format, which was stressed in the verbal discussions, while others were stimulated by the formality of the procedure which was stressed in the written materials. The three-stage organization of the format allows the very unmotivated students to be detected early so that pressure can be applied.

In summary, the project format described in this paper has been successful in introducing the students to the experience of being a scientist. Anyone interested in receiving a sample copy of the journal from this year's course can do so by writing the author.

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Driven "portulum": A rolling ball as a simple oscillating system

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A classroom demonstration, a variation of the simple swinging pendulum, is described. In our "portulum," a ball, driven by short blasts of air, rolls along a curved tube. The design of this device, its construction, and its usefulness to the teaching of physics are discussed. It is also shown that the oscillations of the rolling ball have the same mathematical form as the oscillations of a ball swinging along the same path, but with a lower frequency.

I. INTRODUCTION

The pendulum is the warhorse of the physics teacher, his favorite example of a simple oscillating system. Though the details of its behavior are actually far from simple and have occupied physicists and mathematicians since at least the time of Galileo,[1] it is a rare student of elementary mechanics who does not receive his first introduction to periodic motion by observing a hanging mass swinging in front of the classroom.

In the following article we describe a variation on this familiar theme: a rolling ball, driven by short blasts of air, moves along a curved tube in simulation of the motion of a swinging mass. Since the mass in our apparatus is not suspended (i.e. "hanging under") but supported (i.e., "borne upon") it should perhaps be more properly called a "portulum" than a pendulum.

The portulum we describe here is relatively simple to construct and provides a striking classroom demonstration of both oscillatory motion and mechanical resonance. Unlike the simple pendulum, its mass may easily be constrained to move in paths other than a circle, especially for demonstrations of cycloidal or "isochronous" motion. The device also may be easily adapted to display the ball's motion quantitatively by using photoelectric sensors mounted along its path to trigger timers.

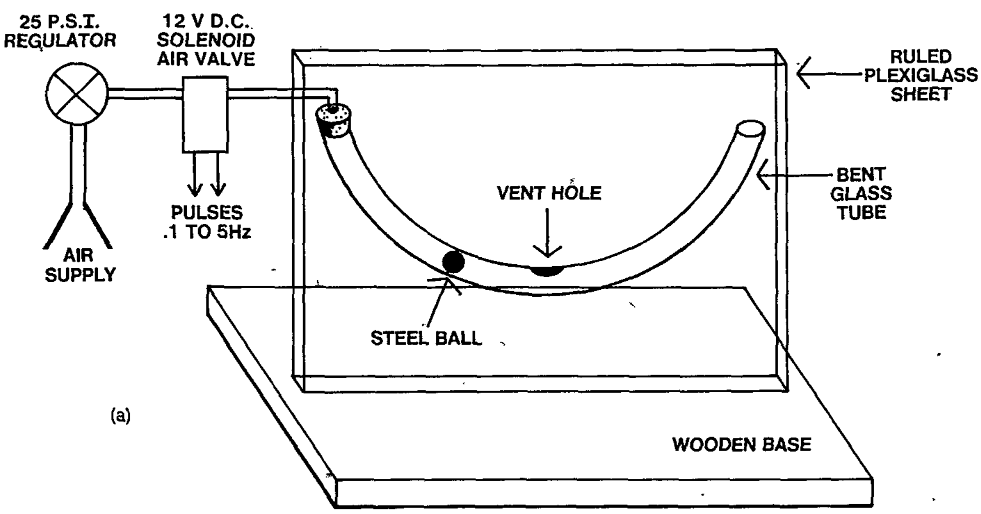
In Sec. II we describe the design and construction of our driven portulum. Section III discusses the motion of the rolling ball and demonstrates that this motion has the same mathematical form as the motion of a mass swinging along the same path, but with a lower frequency. Finally, Sec. IV points out some possible applications of the apparatus both to classroom teaching and to laboratory experiments.

II. CONSTRUCTION AND DESIGN

A sketch of the mechanical system of our demonstration apparatus is shown in Fig. 1(a). A $\frac{3}{8}$-in.-diam steel ball rolls inside a bent glass tube of inside diameter slightly larger than the ball. The tube is mounted on a ruled plexiglass sheet which is fixed vertically on a wooden base, so that the entire apparatus may be conveniently set on a desk or laboratory table.

One end of the glass tube is open. The other is plugged with a small stopper through which we insert a thin copper nozzle which is connected to a solenoid valve which controls the flow of air from a compressed air line or a demonstration gas bottle. The solenoid valve used in our apparatus can only seal pressures below 45 psi, so a 25-psi regulator was placed between the valve and the air line.[2] The valve on this regulator also allows adjustment of the magnitude of the driving force on the ball so that it is not shot out of the open end of the tube.

The motion of the ball is driven by short blasts of air generated by electrical pulses fed to the solenoid valve. The electrical system to do this is outlined in Fig. 1(b). For a circular tube of 1 m radius of curvature, the natural oscillation period of the ball is about 2 sec [see Eq. (3) in Sec. III]. We therefore drive the solenoid with pulses derived from a Hewlett-Packard Low Frequency Function Generator. The square-wave output of this device is used to



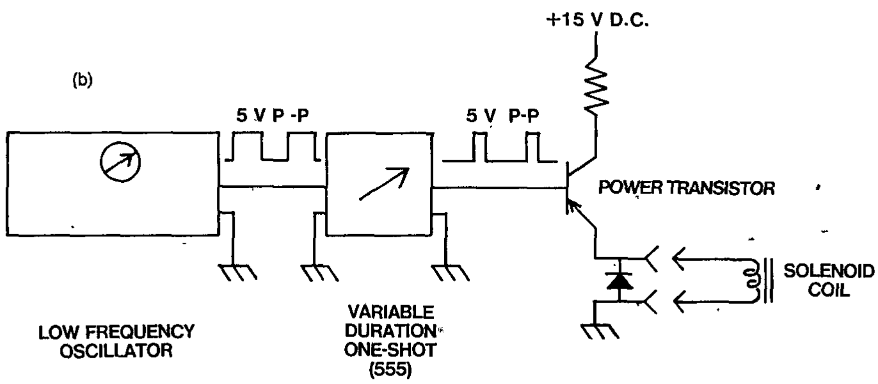


Fig. 1. (a) Mechanical design of the portulum and (b) electronic circuitry of the portulum.

trigger a variable one-shot (constructed from the ubiquitous 555 timer chip), which is adjusted to provide pulses of short duration relative to the square-wave period. The output from the one-shot is connected to the base of a power transistor which drives the solenoid. Details of the circuit may vary depending on the type of valve used; a schematic of the circuit in operation with our particular device is available from the author.

The duration and frequency of the pulses can be set to provide optimum behavior of the rolling ball. If the pulses are short enough and if there is rapid leakage of air around the edges of the ball, it will behave as if the driving impulse were applied instantaneously. The actual driving function of our portulum is more complex, of course, since at least some of the air introduced by the valve continues to expand behind the ball as it rolls to the right (and the ball compresses air ahead of itself as it rolls to the left, introducing a damping force).

To minimize this effect, a vent hole is drilled through the upper surface of the curved tube at its lowest point [see Fig. 1(a)]. Once the ball rolls past this point to the right, excess pressure is vented and the ball continues its motion as a relatively freely rolling mass. If desired, the vent hole could be drilled closer to the air input so that a larger fraction of the ball's motion could be simplified. For purposes of demonstrating mechanical resonance, however, we like to start the ball from rest at the bottom of the tube; this would not be possible if the vent hold were moved to the left.

Several glass tubes bent to different shapes can be provided for the apparatus. We initially experimented with clear plastic tubes used for chemistry and biology labs, but found that their inner surfaces were not uniform enough for smooth motion of the ball. Glass tubes, however do have a disadvantage of being too smooth inside to insure that the ball rolls without slipping. Etching the lower inside surface of the tube by running a drop of hydrofluoric acid along it may be a remedy to this problem.

In operation, the device is simply connected to the air lines, the electronics are turned on, and the frequency of the pulses is slowly varied until the ball rolls noticeably back and forth. The regulator valve can be used to adjust the amplitude of the driven oscillations.

For small classrooms the device can be viewed directly. In larger classes, the device could be backlit through the plexiglass or even shadowed on a wall by a lamp placed in front of it. It could also be scaled up somewhat, although higher pressures might be needed to drive heavier balls.

III. BEHAVIOR OF THE ROLLING BALL

Our apparatus differs from a conventional pendulum in that our ball is not suspended by a wire or rod but rolls along a track. This method of constraint transfers some of the energy of translation of the ball into energy of rotation. Is the general behavior of the rolling mass similar to that of the suspended mass? We consider two ideal cases.

A. Circular motion

It is not difficult to show that if the track on which the ball rolls is circular, its motion is analogous to that of a swinging mass; indeed this is a common supplementary problem in many intermediate mechanics textbooks.[3] Referring to Fig. 2, where R is the radius of the circular path,

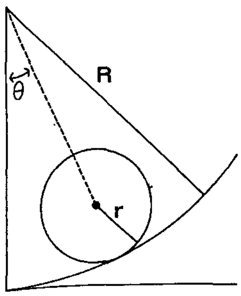


Fig. 2. Coordinate system to describe a ball rolling on a circular track.

r is the radius of the rolling ball, and θ is the angle of deflection from the vertical, an equation of motion of the ball (assuming it rolls without slipping) is given by

$$\ddot{\theta} + \frac{5}{7}\frac{g}{R-r}\sin\theta = 0. \tag{1}$$

This motion is not simple harmonic, but for small amplitude oscillations we can approximate the motion by

$$\ddot{\theta} + \frac{5}{7}\frac{g}{R-r}\theta = 0, \tag{2}$$

which represents simple harmonic motion with a frequency of

$$f = \frac{1}{2\pi}\sqrt{\frac{5}{7}\frac{g}{R-r}}\,. \tag{3}$$

For a swinging mass, the analogous equations are given by

$$\ddot{\theta} + (g/L)\sin\theta = 0, \tag{4}$$

$$\ddot{\theta} + (g/L)\theta = 0, \tag{5}$$

and

$$f = (\tfrac{1}{2}\pi)\sqrt{g/L}, \tag{6}$$

where $L = R - r$, see Fig. 2. Therefore if the ball rolls along a circular path it executes the same type of motion as a swinging one, but with a frequency that is smaller by the ratio $\sqrt{5/7}$.

B. Cycloidal pendulum

It is well known that the motion of a swinging pendulum is not precisely isochronous [as can be proven by solving the exact equation of motion, Eq. (4) above].[4] One of the first lab experiments conducted in the introductory classes at our school, for instance, demonstrates that the period of oscillation increases noticeably with increasing amplitude. A pendulum travelling in a cycloidal path, however, will display isochronicity. This fact was first realized by Huygens in 1673 who constrained a pendulum to a cycloidal path by forcing it to swing between a pair of cycloidal walls.[5]

A rolling ball may more easily be constrained to follow a cycloid. In our apparatus the glass tube which contains the ball is simply formed to the proper shape. The following argument, which is far less common than the one for the circular pendulum, demonstrates that a ball rolling along a cycloidal path without slipping does indeed execute simple harmonic motion and is therefore isochronous.

Referring to Fig. 3, we assume that the center of the ball moves along a path S and the ball itself along the track S'. The shape of S' will, for the moment, be considered arbitrary, but since the ball is always in contact with S', the perpendicular distance between S' and S is r, the radius of the ball.

We may then write

$$x = x' - r\sin\beta, \tag{7a}$$

$$y = y' + r\cos\beta, \tag{7b}$$

where (x,y) and (x',y') are the coordinates of the center and contact points of the ball and β is the angle between the tangent to S' at (x',y') and the horizontal.

Differentiating, we find

$$dx = dx' - r\cos\beta\, d\beta, \tag{8a}$$

$$dy = dy' - r\sin\beta\, d\beta, \tag{8b}$$

from which we can easily derive an expression relating the distance ds travelled by the center of the ball as it rolls along a length of track ds':

$$\begin{aligned} ds^2 &= dx^2 + dy^2 = dx'^2 + dy'^2 \\ &\quad - 2r\,d\beta(dx'\cos\beta + dy'\sin\beta) + r^2 d\beta^2 \\ &= ds'^2 - 2r\,d\beta(dx'\cos\beta + dy'\sin\beta) + r^2 d\beta^2. \end{aligned} \tag{9}$$

Now since

$$\cos\beta = \frac{dx'}{ds'} \quad \text{and} \quad \sin\beta = \frac{dy'}{ds'}, \tag{10}$$

we may write Eq. (9) as

$$ds^2 = ds'^2 - 2r\,ds'\,d\beta + r^2 d\beta^2 = (ds' - r\,d\beta)^2 \tag{11}$$

or

$$ds = ds' - r\,d\beta \tag{12}$$

and the speed of the center of the ball is given by differentiating Eq. (12) with respect to time:

$$\dot{s} = \dot{s}' - r\dot{\beta}. \tag{13}$$

We also define ψ, the angle between the point of contact of the ball and the track and a fixed point on the ball, A, which is in contact with the track at its lowest point (see Fig. 4). Since the ball rolls without slipping, we know that $\dot{s}' = r\dot{\psi}$ and we may use Eq. (13) to write

$$\dot{\psi} = \dot{s}/r + \dot{\beta}. \tag{14}$$

We now proceed to derive an equation of motion for the rolling ball from energy considerations. The translational kinetic energy of the ball is given by

$$E_{\text{trans}} = (\tfrac{1}{2})m\dot{s}^2 \tag{15}$$

and the rotational kinetic energy of the ball is given by

$$E_{\text{rot}} = (\tfrac{1}{2})I\omega^2 = (\tfrac{1}{2})I(\dot{\psi} - \dot{\beta})^2. \tag{16}$$

For a uniform ball the moment of inertia about the center, $I = (\frac{2}{5})(mr^2)$. Substituting from Eq. (14) and simplifying

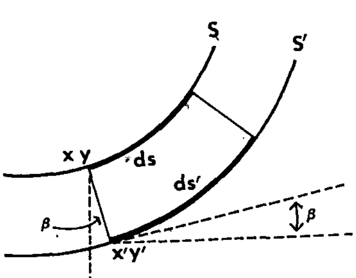


Fig. 3. Coordinate system to describe a ball rolling on an arbitrary track.

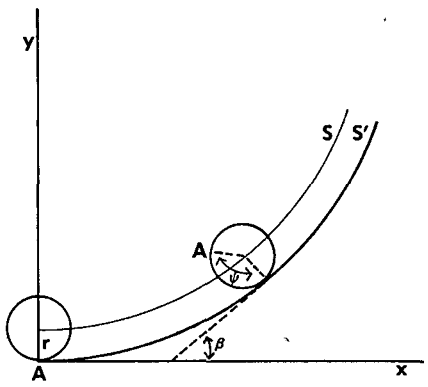


Fig. 4. Coordinate system to describe a ball rolling on a cycloidal track.

we may finally write

$$E_{\rm rot} = (\tfrac{1}{5})m\dot{s}^2. \quad (17)$$

The total kinetic energy, adding Eqs. (15) and (17) is thus

$$E_{\rm kin} = E_{\rm trans} + E_{\rm rot} = (\tfrac{7}{10})m\dot{s}. \quad (18)$$

Our derivation of the kinetic energy of the ball makes no reference to the shape of its path. To specify its potential energy we assume that the path S of the ball's center is specifically a cycloid given by the parametric equations (referring again to Fig. 4):

$$x = a\theta + a\sin\theta, \quad (19a)$$

$$y = a(1 - \cos\theta), \quad (19b)$$

where θ is a variable parameter (not a polar angle) and $2a\pi$ is the distance between the cusps of the cycloid. The element of path length is then

$$ds = \sqrt{dx^2 + dy^2} = 2a\cos\phi\, d\phi \quad (20)$$

and by integration we find

$$s = 4a\sin\phi, \quad (21)$$

where $\phi = \theta/2$.

Now the potential energy of the ball is given by

$$E_{\rm pot} = mgy = mga(1 - \cos2\phi) = (mgs^2/8a), \quad (22)$$

and the total energy of the rolling ball, from Eqs. (18) and (22), is

$$E_{\rm tot} = E_{\rm kin} + E_{\rm pot} = (\tfrac{7}{10})m\dot{s}^2 + (g/8a)ms^2. \quad (23)$$

Since this total energy is conserved, we can derive an equation of motion for the rolling ball by differentiating Eq. (23) with respect to time and setting the derivative equal to zero:

$$dE_{\rm tot}/dt = 0 = (\tfrac{7}{10})m(2\dot{s}\ddot{s}) + (g/8a)m(2s\dot{s}), \quad (24)$$

which readily reduces to

$$\ddot{s} + (5g/28a)S = 0. \quad (25)$$

This is the equation of motion of a simple harmonic oscillator. Clearly the rolling ball is isochronous, just like the swinging one, but with a frequency of

$$f = (\tfrac{1}{2}\pi)\sqrt{(\tfrac{5}{28})(g/a)}, \quad (26)$$

which is $\sqrt{\frac{5}{7}}$ as great as the frequency for a swinging cycloidal pendulum.[6]

We point out again that the derivation of the kinetic equation of the ball, Eq. (18), is made with reference to the shape of the track along which the ball rolls as long as the radius of curvature of the track is larger than that of the ball and as long as the ball moves without slipping. Thus only the potential energy function, Eq. (22), depends on the shape of the path (the functional relation between y and s). In short, whatever the shape of the path of our rolling ball, it will simulate the motion of a ball swinging in the same path (or sliding along a frictionless wire), but with a lower frequency.

C. Nonideal cases

Slipping of the ball may introduce measureable differences between ideal and real performance for our apparatus. Differences between the aerodynamics of the ball in the tube and the freely swinging ball may also produce discrepancies between the two cases. Since we designed our apparatus primarily as a classroom demonstration of oscillatory motion and mechanical resonance we have not fully investigated these problems. The investigation of these deviations may provide a challenging research project for upper-class physics majors.

IV. APPLICATIONS

The driven portulum we have described here was originally conceived with two applications in mind. First, it was desired to demonstrate the effect of different-shaped paths on the motion of an oscillating mass. This can more easily be done with our device than with a conventional swinging pendulum since one need merely rebend the constraining tube to a different curve to alter its path. Huygens's method of "constraining walls" for altering the path of a swinging pendulum is clearly more difficult to implement in practice and it is more difficult for students to visualize the resulting path of motion. (They must learn that the "involute" of a cycloid is another cycloid.)

Since, as we have seen, the rolling ball behaves like a more slowly swinging mass, the portulum can thus be used as a versatile substitute for a less easily adjustable suspension system. It can also be presented as a variation of the swinging pendulum and the behavior of the two systems can easily be compared: the longer period of the rolling ball can be measured simply by recording the frequency setting of the oscillator required for maximum amplitude oscillations.

Secondly, we wished to construct an improved device for illustrating mechanical resonance to introductory classes. The apparatus most familiar to us for this purpose is a mass hung from a spring which is in turn attached to a rod that can be driven up and down by a variable-speed motor. As the speed of the motor is varied, the mass is seen to execute the greatest oscillations when the period of the driving motor is matched to the natural period of the spring–mass combination (which may of course be measured by bouncing the mass with the motor turned off). The pneumatic driving mechanism we use is fundamentally no less simple than this system and, we believe, its operation is more transparent to a classroom. Students watching a mechanically driven oscillator cannot see the relatively small motions of the driving rod, nor can they read the speed dial on the motor. Our pneumatic portulum, on the other hand, provides a class with an audible cue, the hiss of escaping gas, when the driving force is applied. An LED connected to the solenoid-valve driving circuit can also provide a visual cue.

In operation, of course, the demonstration apparatus is used just like its mechanically driven counterpart. The natural frequency of the ball is determined by rolling it in the tube without a driving force. The period of the driving oscillator is then adjusted to produce maximum-amplitude oscillations.

The device we have described has met our two major objectives, but its usefullness may extend beyond the demonstration cart. For more quantitative measurements small photoelectric detectors can be mounted on the plexiglass mounting sheet to trigger a timer when the ball passes by.[7] Thus the motion of the rolling ball can more easily be monitored than the corresponding motion of a swinging one. This setup would make our device useful as an undergraduate laboratory experiment.

Ultimately, we believe, the general design of the portulum could be modified to suit many purposes, since the rolling ball may more easily be constrained and monitored than a swinging one. For instance the glass tube itself could be evacuated and sealed off to minimize air resistance and the ball could be driven by electromagnets triggered by photocell gates mounted on the plastic sheet. We stress here, however, that the form of the device described in Sec. II offers a simplicity of design and construction and a transparency of operation that is a distinct advantage in the general physics classroom. Whatever limitations it has, such as departures from ideal behavior due to friction and pneumatic effects, it also shares with its more conventional counterparts.

ACKNOWLEDGMENTS

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Resolving time effect on counting statistics

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The observed variance in the counts recorded from a radioactive source is related to the response time of the detector, in a way which is readily exploited as an intermediate laboratory exercise.

A fundamental limitation[1,2] on the operation of virtually every form of particle detector or counting system is a characteristic minimum response time, or "dead time." This is the time that must elapse, after the detector has responded to an event, before it is capable of an equivalent reponse to a subsequent event. The dead time may result from physical processes in the detector itself (the clearing of positive ions after a Geiger discharge), or from characteristics of the associated electronics (analog-to-digital conversion in a multichannel analyzer), or from a combination of both. In any case the effect is that, except in the limit of very low counting rate, some events go undetected, and the response of the detector to the true event rate is nonlinear. Physics students usually first encounter the concept of dead time, or resolving time, in an intermediate or advanced undergraduate laboratory.

If one wants to measure the dead time of a detector as a laboratory exercise, the best choice of detector is almost certainly the Geiger tube. It is simple, inexpensive, efficient, and slow enough (typically around 10^{-4} sec) so that response-time effects are easy to observe. The usual method for inferring the dead time from counter performance is the *split-source method*, which relies on the fact that the count rate one observes from a source is less than the sum of the count rates observed separately from its parts, because a higher proportion of counts are lost at the higher count rate. (A typical example can be found in the lab manual issued by Ortec,[3] Experiment 2.4.) A less familiar effect of finite detector resolving time is that the *statistical fluctuations* of the observed counts are reduced when dead-time losses are significant. This phenomenon can also be exploited in a laboratory exercise for undergraduates.

As the simplest model of a system with finite response time, consider a nonparalyzable detector with a fixed response time τ. Each event to which the detector responds is assumed to initiate a time interval of length τ, during which the detector is wholly unresponsive to further events. This model gives the same first-order effects as would a

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