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Hex

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Hex

Abstract

The game hex has simple rules but complex game play. A computer program was used to analyze different boards. This yielded an interesting insight in making the optimal Hex program.
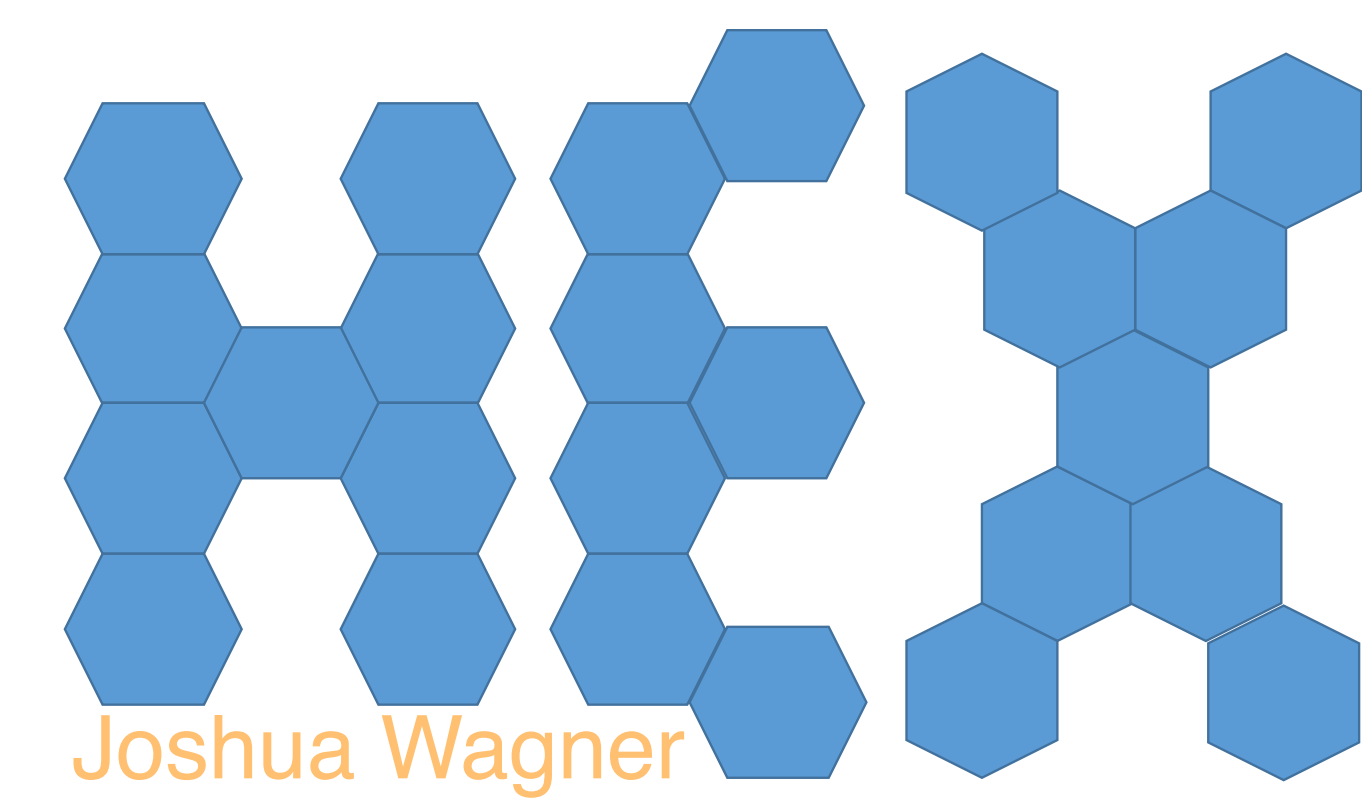
Keywords

Hex Game, Monte Carlo

Disciplines

Computer Sciences

Comments

This poster was made for Professor Todd Neller's First Year Seminar, *FYS 187-4: Games and Computation*, Fall 2015. It was presented as part of the first CAFE Symposium, 2016.

HEX

Joshua Wagner

What is Hex?

Hex is a game played on a rhombus shaped board covered in a hexagonal grid of cells.

This game is considered a combinatorial game. A combinatorial game is a game with no elements of chance and no hidden information.

Hex is further defined as a connection game. A connection game is a subcategory of combinatorial games in which the goal is to form some type of connection across a board.

Gameplay

Hex gameplay is simple.

Players alternate placing pieces of their own color on the board.

A player wins after connecting both their sides of the board.

The four corners are connected to both players' sides.
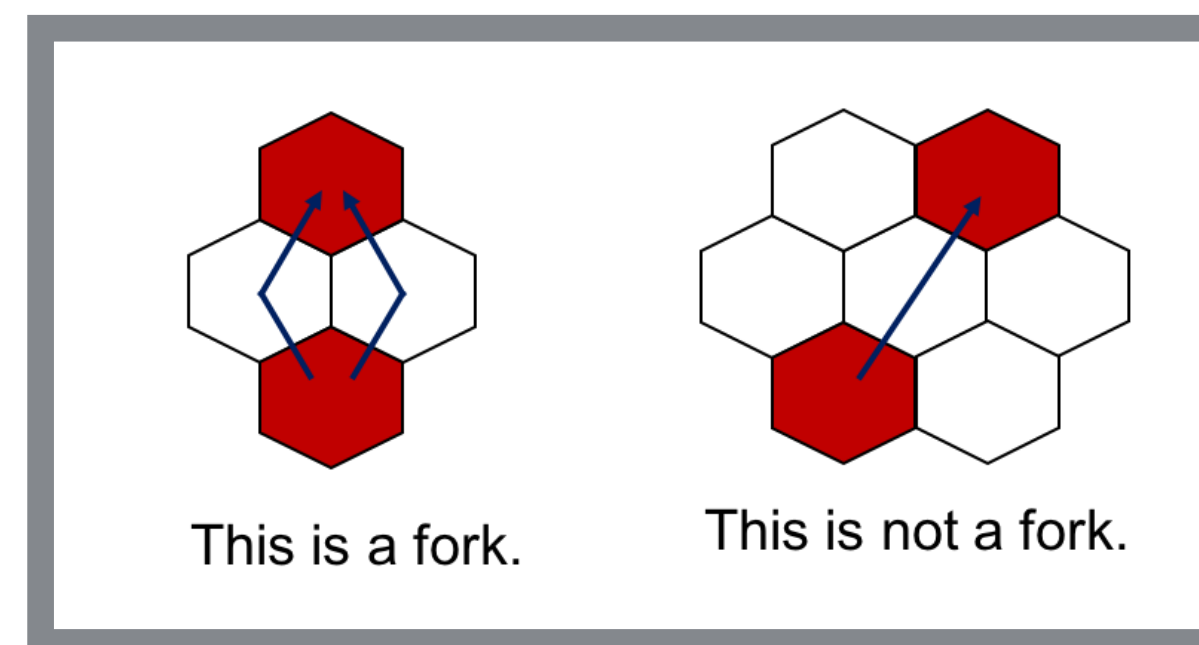
Larger board sizes create increasingly complex play.

An Important Structure

The fork is the most important piece formation in the game of Hex.

A fork poses two possible potential connections between the same two pieces.

With perfect gameplay, it is impossible for an opponent to cross a fork.

There are a few examples below.

This is a fork.   This is not a fork.

This is a small blank board. Notice that A1,E1,A4, and D4 are connected to both sides.

In this example green wins.

Professor Todd Neller provided the mentorship and computer program necessary to carry out this research.

With perfect play, green will win. Green has created a series of forks across to board and has essentially won.

With perfect play, red will win. Red is one move away from completing a series of forks, and green has only a single connection between each of green's pieces.
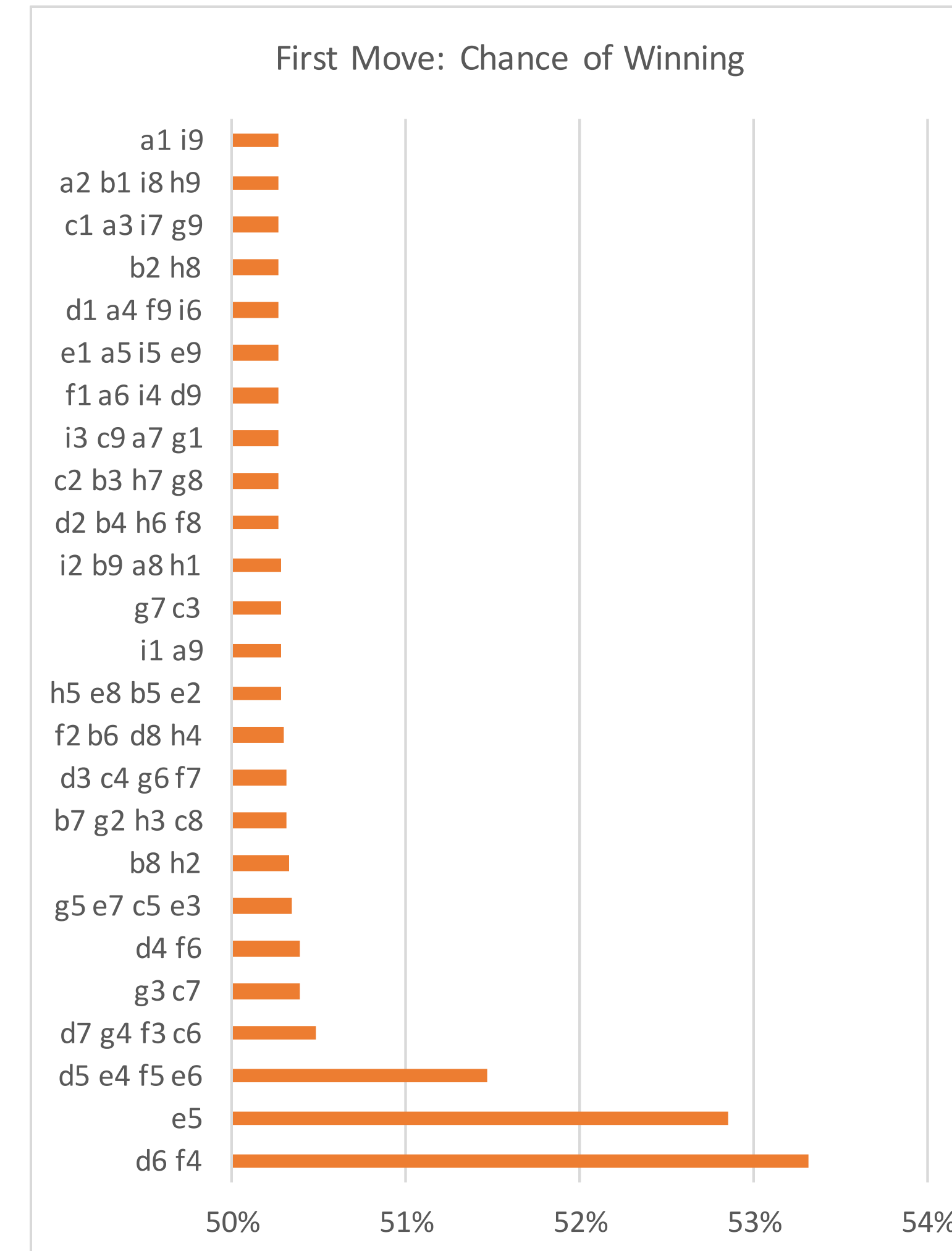
First-Player Advantage

There is a distinct advantage for the first player in Hex. Plays in the center of the board are often considered stronger. Certain rules have been put in place to eliminate the first-player advantage such as the swap rule.

A Monte Carlo program was run five times, and the chance of winning with each first move was found. Board symmetry was accounted for as well.

It is important to note that this estimate is based on a Monte Carlo program which relies on a vast number of randomly played games to estimate cell values.

First Move: Chance of Winning

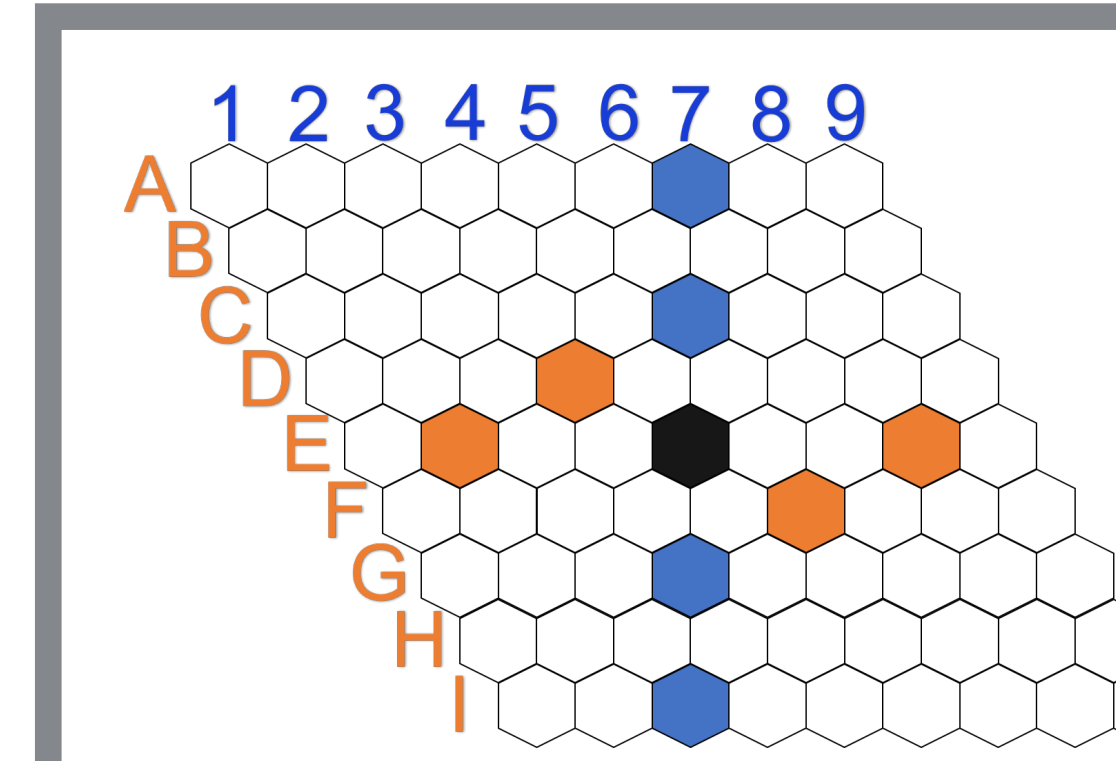| Cell	Chance of Winning
d6 f4	0.53305720
e5	0.52842300
d5 e4 f5 e6	0.51458105
d7 g4 f3 c6	0.50475990
g3 c7	0.50390020
d4 f6	0.50387660
g5 e7 c5 e3	0.50343355
b8 h2	0.50326270
b7 g2 h3 c8	0.50313875
d3 c4 g6 f7	0.50311560
f2 b6 d8 h4	0.50286940
h5 e8 b5 e2	0.50275145
i1 a9	0.50273150
g7 c3	0.50271720
i2 b9 a8 h1	0.50270070
d2 b4 h6 f8	0.50267480
c2 b3 h7 g8	0.50266550
i3 c9 a7 g1	0.50266320
f1 a6 i4 d9	0.50265390
e1 a5 i5 e9	0.50264650
d1 a4 f9 i6	0.50264070
b2 h8	0.50263430
c1 a3 i7 g9	0.50263275
a2 b1 i8 h9	0.50262785
a1 i9	0.50260320

Multiple cells have the same value due to board symmetry.

The moves are ranked 1-25. 1 being the best moves and 25 being the worst moves.

```
  1 2 3 4 5 6 7 8 9
A 25 24 23 21 20 19 18 15 13
B 24 22 17 16 12 11 9 8 15
C 23 17 14 10 7 4 5 9 18
D 21 16 10 6 3 3 4 11 19
E 20 12 7 3 2 3 7 12 20
F 19 11 4 3 3 6 10 16 21
G 18 9 5 4 7 10 14 17 23
H 15 8 9 11 12 16 17 22 24
I 13 15 18 19 20 21 23 24 25
```
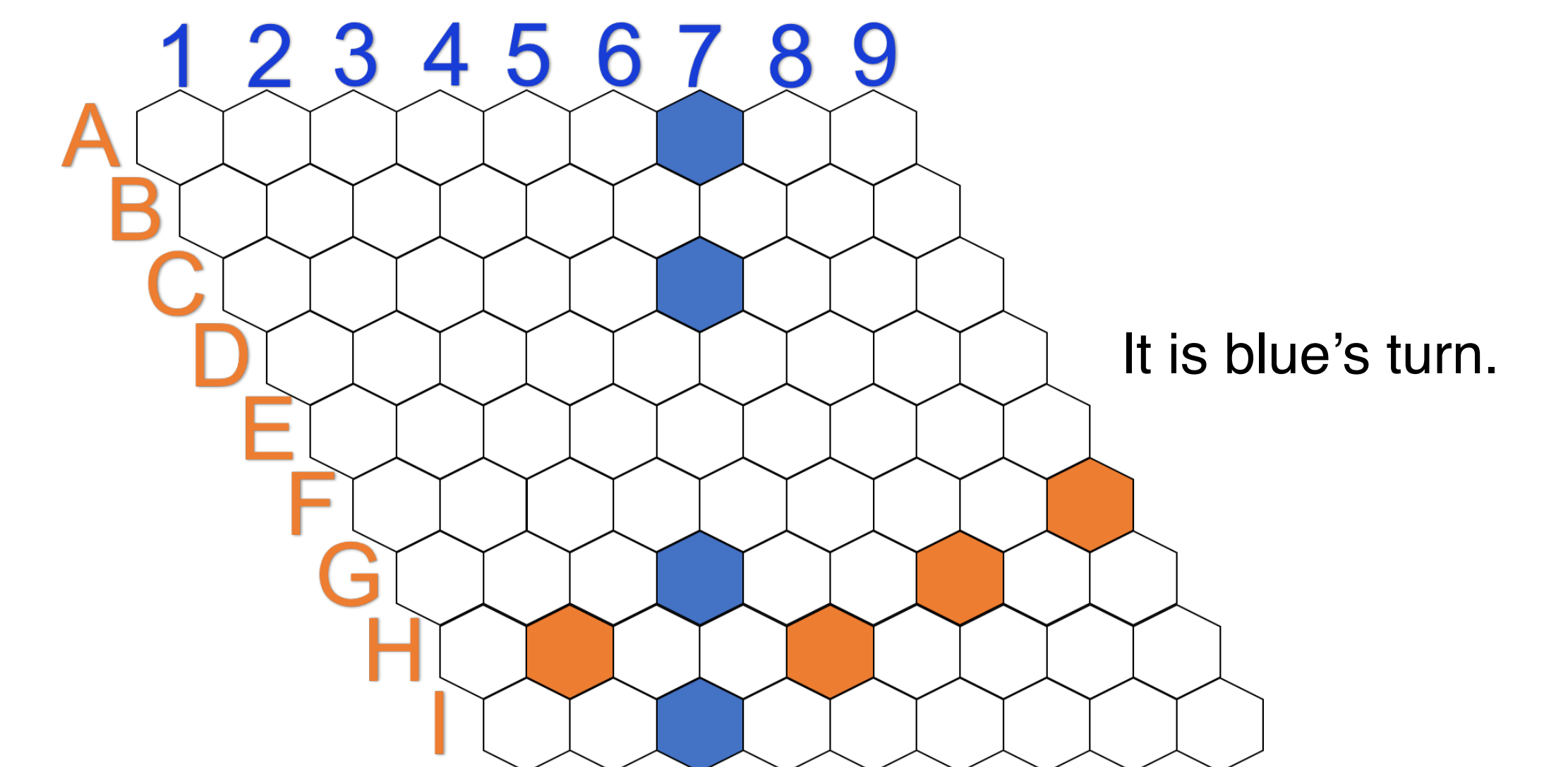
Too Easy

As mentioned, the Monte Carlo program uses randomly played games to evaluate cell values. Each cell is played a certain number of times at the beginning of the program. These initial plays enhance the program's exploration.

The number of games evaluated and the number of exploration games can be changed to optimize the program.

The board below was used to test the effects of these values on the program's ability to play competitively. The black space is the perfect play for this game.

The program was dependably able to choose e5 (the black space) with incredibly small values. This decision is needed by both orange and blue to complete a series of forks.

Hex is a zero-sum game meaning that when one player loses, another wins. Forming a series of forks and preventing orange from forming a series, practically doubled the utility of playing in the black cell.

A Different Approach

When using a different board setup, the program was much less dependable and chose different cells even when using the same number of games and exploration games.

Averaging the values of 20 different program runs, each playing 100,000 games and 20 exploration games per cell, may give a better estimate of the value of each cell.

As a control, the program was run on the same board. This control played 2,000,000 games and 400 exploration games per cell. In this way, both methods used the same number of games and exploration games per cell.

It is blue's turn.

Interestingly, neither method found the perfect play for this board setup (e5).

Cell	Value	Cell	Value
h3	0.5841855	f4	0.783188
h4	0.5368129	h3	0.514103
f4	0.55129835	h4	0.514103
f5	0.5499466	e4	0.505945
e5	0.5514005	e5	0.505945
g3	0.52237825	f3	0.503768
g6	0.51145955	f5	0.503768
e6	0.5245535	d5	0.503661
d6	0.5253916	c5	0.503661
d5	0.52414795	d4	0.503656

These are the top ten moves gathered from 20 iterations of the Monte Carlo program with 100,000 games and an exploration of 20.
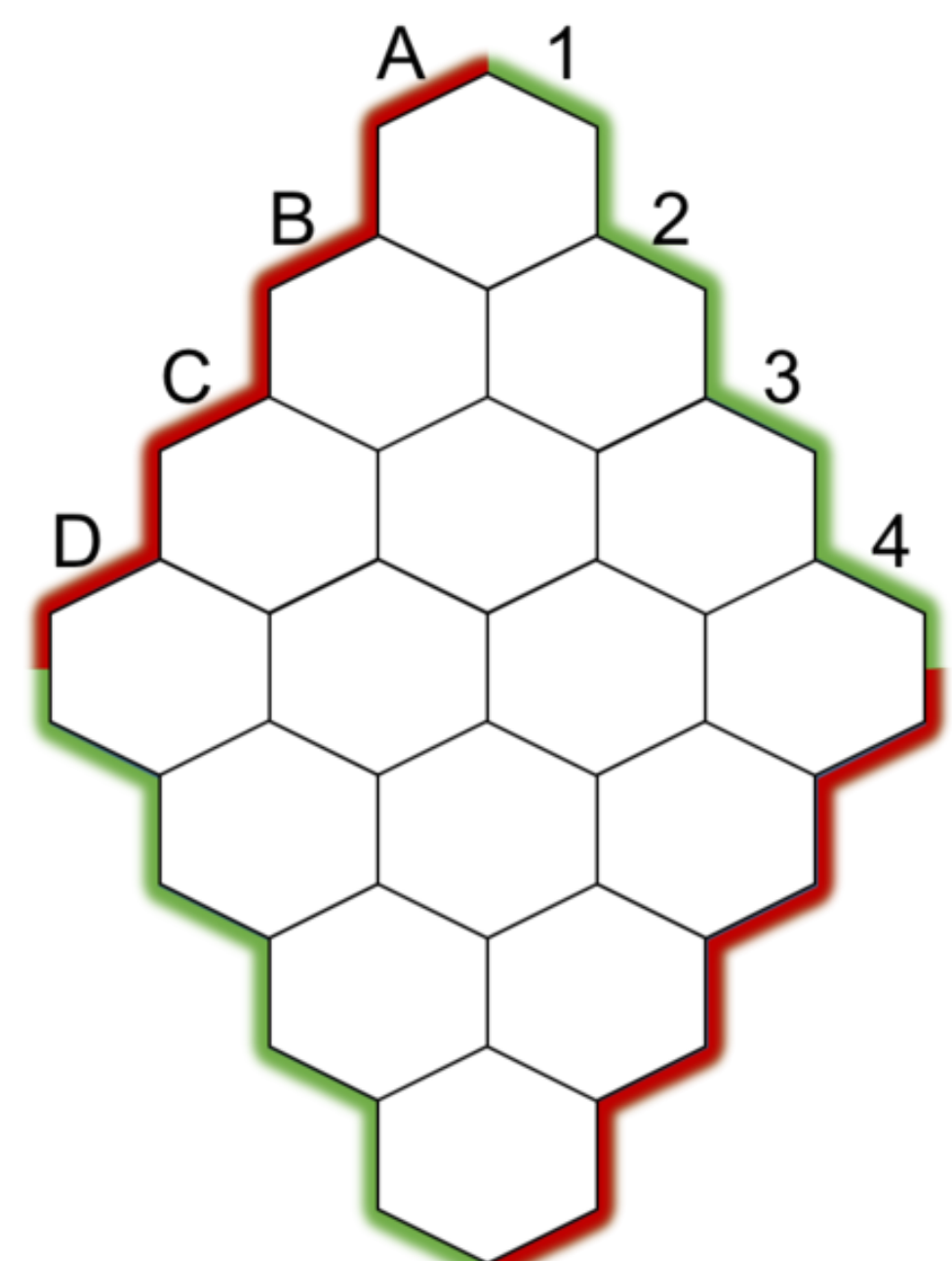
These are the top ten moves gathered from 1 iteration of the Monte Carlo program with 2,000,000 games and an exploration of 400.

f4 may lead to a win through perfect play, but the 20-game average's choice of h3 is a wasted move because it is just completing a fork.
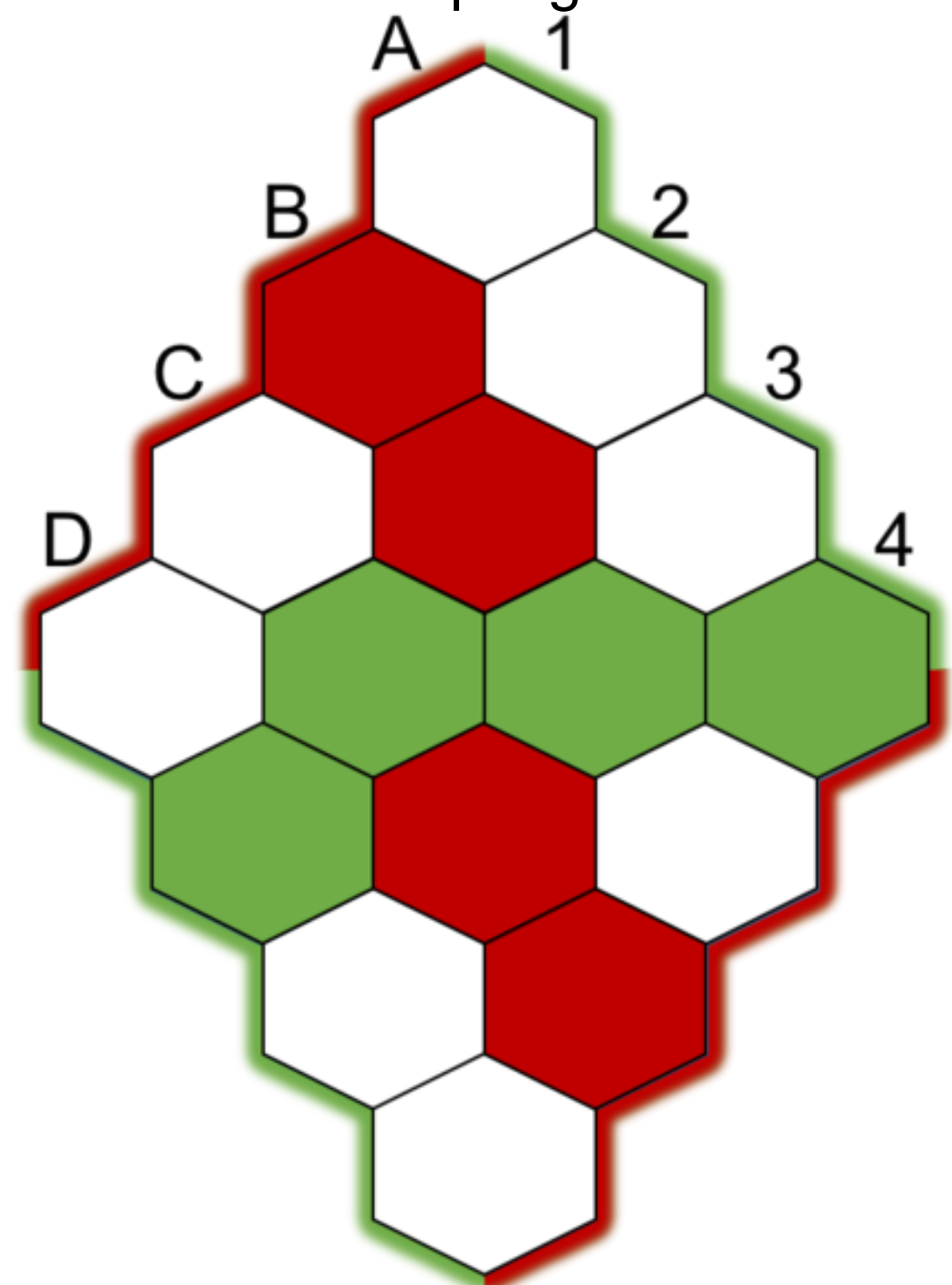
Two of the 20 games found that e5's value was .961028 and .960458. These values are desired, as e5 is the perfect play for this board setup. The remaining eighteen games' averages collectively brought down these high scores.

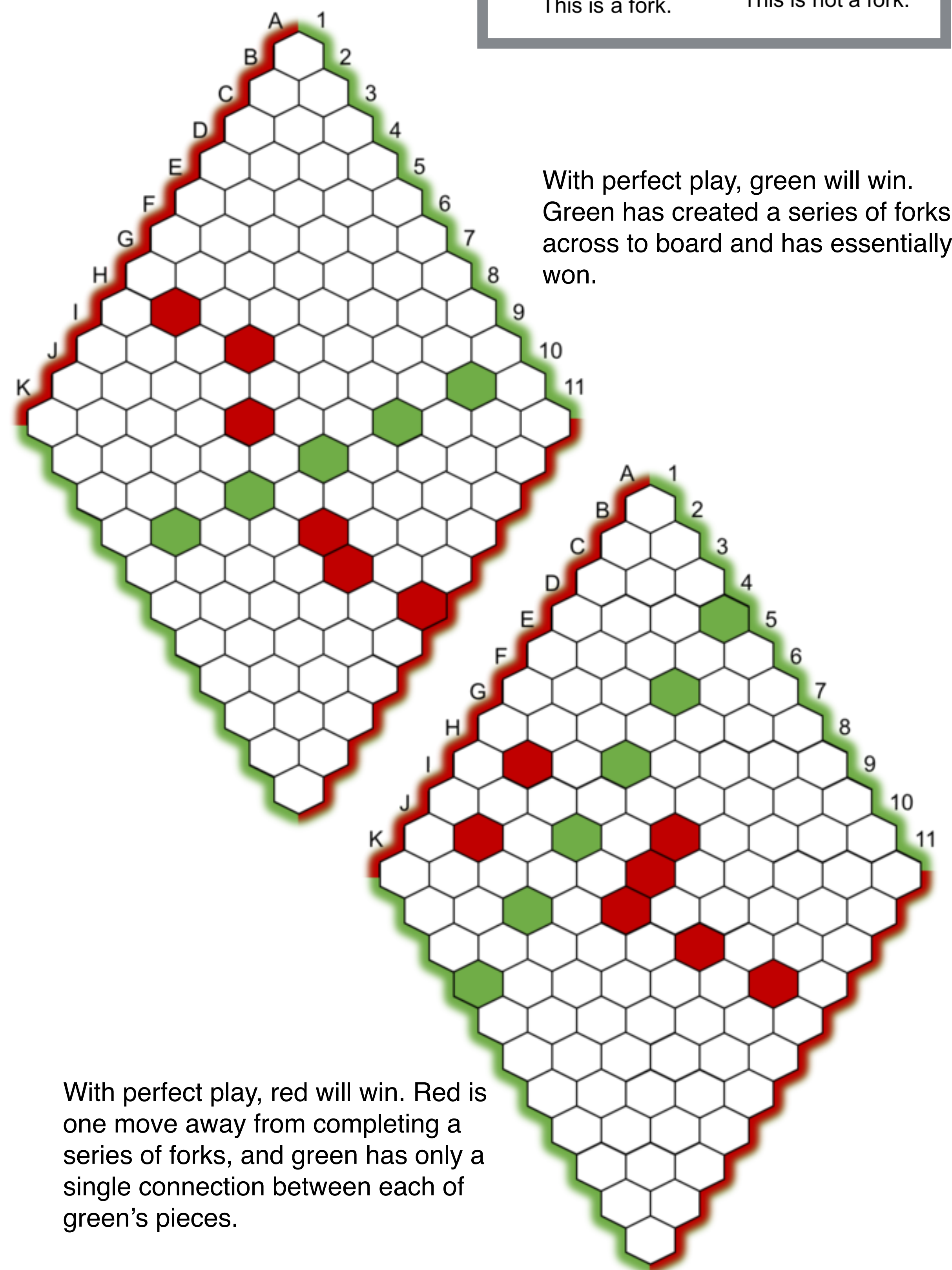Perhaps an ideal program would play smaller games until a cell choice resulted in a probability of winning above some threshold near 0.96.