Price Barriers in the Stock Market and Their Effect on the Black-Scholes Option Pricing Model

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By Nathan Blyler

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I. Introduction

A. Financial Background

The option market is closely related to the stock market in many ways, but differs in trading practices and uses. Similar to the stock market, the options market relies on speculation of stock price movement over time, but unlike a stock the derivative is only valuable over a set period of time. In this paper we will focus on call options. A call is simply the right to buy a security at a specified price called a strike price. The writer of a call is agreeing to sell the security at the strike price when the buyer exercises the call before a set date. The buyer can choose to let the call expire without ever exercising it, which would normally be done if the price of the security falls below the strike price. Thus, the buyer could simply buy the stock at market price for less than the strike price. In the
case where the strike price is above market price, the call is said to be out-of-the-money, and when the strike price is below the market price, the call is in-the-money. Since an option can move into the money at any time, the price of the option must not only be the difference between the strike price and the current security price to account for immediate exercising (if the call is in-the-money and zero otherwise) but also include a time premium for the chance a call moves to being in-the-money.

Options are appealing to investors for numerous reasons. They are substantially cheaper than buying a security, and are normally settled in cash rather than trading the underlying security at the specified price. This allows investors to speculate on a security’s price movement with less initial capital and realize larger percentage returns on their investments. Furthermore, the options market allows for speculation on the volatility of a security’s price rather than the direction of its movement. The final use of the options market is to hedge risky positions taken in the stock market. Consider someone who has sold a stock short without owning the stock and has unlimited loss potential. However, if she buys a call option giving her the right to buy the stock at a given price, she can only lose the premium she paid and the difference between the strike price and the price at which she short sold the stock. The use of call options as in this manner led to the most well known option pricing method called the Black-Scholes Model.

In their ground-breaking paper on corporate liabilities, Black and Scholes created a model for pricing call options based on hedging to form a riskless portfolio of stocks and options (Black and Scholes 1973). One of their assumptions was no arbitrage in the market, so any risk-free portfolio should provide a return at the risk free rate, allowing them to find a price for call options given the following inputs: security price, strike price, risk free rate, time to expiration, and volatility of the underlying security’s returns. Their predicted
The price of a call option is found by the following system of equations, which can be thought of as the risk adjusted probability an option finishes in the money:

\[ C_0 = S_0 \Phi(d_1) - X e^{-rT} \Phi(d_2) \]  

(1)

\[ d_1 = \frac{\ln(S_0/X) + (r + \sigma^2/2)T}{\sigma \sqrt{T}} \]  

(2)

\[ d_2 = d_1 - \sigma \sqrt{T} \]  

(3)

where

- \( C_0 \) = Current call option value
- \( S_0 \) = Current stock price
- \( \Phi(d) \) = The probability a random draw from a standard normal distribution will be less than \( d \).
- \( X \) = Exercise price
- \( r \) = Annualized risk-free interest rate
- \( T \) = Time to expiration in years
- \( \sigma \) = Volatility of the underlying stock

Part of the reason for the widespread use of this model is the ease with which most of these inputs can be found, as all are easily observable except for volatility. Therefore, pricing an option becomes a question of how much the underlying stock’s price will change during the life of the option. The higher the volatility, or movement in price, the higher the probability the stock’s price will end above the strike price.

The model is built on other assumptions besides no arbitrage, such as stock prices follow a Brownian motion, disallowing jumps and predictability. Furthermore, the underlying stock is assumed to pay no dividends and the option is assumed to not be exercised until expiration. Additionally, the model assumes there are no transaction costs in the option market. The most important assumption is the variance of the rate of return of the security is constant. While most of these assumptions obviously do not hold in any market, it is important to note that these assumptions only need to hold over the life of an option, not forever, to make the model prediction valid.
One can easily see, based both on the use of options and the derivation of call pricing, there is a close tie between stock markets and options market. Because of this close relationship, we would expect anomalies in security prices to result in option pricing anomalies. A recent finding in behavioral finance involves the existence of price barriers, such as whole number values, in the stock market (Sonnemans 2003, Dorfleitner et. al. 2009, etc.). These price barriers, or levels of support and resistance, are not unbreakable, but have an effect on the movement of stock prices. As a rising stock price approaches a barrier from below, making that barrier a level of resistance, the volatility of the price usually lessens. If a barrier is approached from above, making it a level of support, the stock price’s rate of change is again likely to slow (Donaldson and Kim 1993).

B. Paper Outline

The stock market is one of the last strong holds for purely rational actors, with many believers in the efficient market hypothesis even while game theory and psychological microeconomics continue to find flaws in the neoclassical assumption of rationality. The efficient market hypothesis does not allow for price barriers to exist in the stock market; therefore, rational investors should not treat options with strikes near the barriers any different than other options. An anomaly around the barriers would indicate that investors who stand to lose substantial amounts of money do not completely believe the efficient market hypothesis.

Comparing option prices to each other does not allow for analysis as they vary in important ways such as distance from the money, time to expiration, and underlying stock. In order to hold these important factors constant and make meaningful comparisons, we use the ‘correct’ B-S predicted price of the option, found by regression techniques. Given the existence of price barriers, the B-S model should misprice options that have a barrier between the stock and strike
price because it does not account for the presence of these barriers. In this paper we will examine errors in the B-S model to detect systematic errors around price barriers.

Our findings show round number price barriers found by other studies have begun to be internalized into the options market. This is strong evidence against the efficient market hypothesis from the actors that are said to be completely rational. Our other proposed price barriers, Bollinger bands and Gann levels, do not act as we hypothesize, but also show investors pay particular attention to specific price levels. Thus, the assumptions made by the B-S model result in other systematic errors not yet discussed in the literature. By combining the behavioral finance literature on price barriers and the literature on errors in the Black-Scholes option pricing model, this paper is the first to look at price barriers’ effect on the options market. This paper strengthens the argument against efficient stock markets by using an efficient option market. Moreover, it adds to the sizable literature on the accuracy of the B-S and how the market price differs from the predicted price in systematic ways.

In the next section, a detailed background of other literature on errors in the B-S option pricing model caused by the model assumptions is given. This is followed by a brief introduction to the discovery of price barriers in stock markets. The following section describes our methodology of finding the correct B-S price and controlling for known systematic errors. Section four justifies the proposed price barriers and the expected findings. Section five gives a summary of the data used, while section six shows the findings. Finally, section seven makes conclusions given the findings and offers alternative hypotheses to explain the unexpected results.
II. Literature Review

A. Black-Scholes Systematic Errors

As with all mathematical models, the assumptions of the B-S model have faced scrutiny from academics since its introduction in 1973. Problems with the assumptions have emerged in varying degrees from unimportant to creating systematic errors in pricing. To avoid misspecification, known systematic errors must be controlled for when examining the difference between the market and B-S price. Theoretically, an American\textsuperscript{1} option is always worth more in the market than exercised, causing many studies to ignore the possibility of early exercising as this violation does not appear to cause systematic errors (Merton 1973, Macbeth and Merville 1979, ect.). Additionally, investors commonly witness jumps in the price of a security from the revealing of new information, violating the assumption of Brownian motion and causing the B-S model to predict prices under the market price (Merton 1976).

The assumption of constant variance in a security’s return throughout the life of an option causes significant disparities between observed prices and predicted prices. A common adaptation has been to model variance as an unpredictable stochastic process (Chesney and Scott 1989, Hull and White 1987, Scott 1987, ect.). The results of modeling stock return variance in this way are inconclusive and make predicting the price of an option considerably more difficult. Hull and White include a volatility of volatility term in their model and find the B-S model to over price at- and in-the-money options; however, they note the over pricing is caused by a positive correlation between the price of the underlying stock and volatility. This is a problem, because the correlation between the two is not constant and has been positive some years and negative others (Rubinstein 1978, Schmalensee and Trippe 1978). Others have tried to

\textsuperscript{1} An American option can be exercised any time between its sale and date of expiration.
model volatility as a function of asset price and time, but failed to achieve better results than attempts to smooth out implied volatility over strike price and time (Dumas et. al. 2002). These failures to create a superior model have allowed the flawed B-S model to remain the standard in option pricing, and as such be using in our analysis of the option market.

Ignoring evidence that stock returns do not have a constant variance over time, simply estimating the expected variance still causes problems as it is the only input not observable in the B-S model. Weighted historical volatility, the simplest of estimators, with recent volatility given the most weight, does not include important factors such as: market volatility, price and volatility correlation, mergers or other large events, and the volatility implied by the options market (Black 1975). In an efficient market, the relevant information would be entirely included in the current market price. Along these lines, multiple authors have found the options market is better at estimating the future volatility of securities returns than historical averages (Black and Scholes 1972, Ncube 1996, Blyler 2012). These studies support our approach to estimating the correct volatility of a stock through the options market.

The largest differences in market and predicted prices occur when an option is far in- or out-of-the-money; however, the direction of these differences is debated. Black finds that far in-the-money options have a market price below the B-S predicted price and far out-of-the-money options have a market price above it (Black 1975). Alternatively, Macbeth and Merville find, on average, in-the-money options are overpriced by the market and out-of-the-money options are underpriced by the market relative to the B-S model (Macbeth and Merville 1979). In partial agreement with the aforementioned authors, Merton finds the market price to exceed the B-S predicted price when the option is both far in- or out-of-the-money and when the option is close to expiration (Merton
1976). Additionally an option’s time to expiration has been found to generate discrepancies between the B-S and market prices. Options with a short time to expiration, three months or less, tend to have a market price greater than predicted by the B-S model (Black 1975). Nevertheless, the extent for which in-the-money and out-of-the-money options are mispriced decreases as time to expiration decreases (Macbeth and Merville 1979). Possible explanations for the systematic error accompany each finding; however, no paper we are aware of mentions the possibility of price barriers in the stock market as a reason for systematic error in the B-S option pricing model.

B. Price Barriers

The study of humans’ psychological ties to specific numbers and the subsequent relation to the stock market is a somewhat recent development. Academic papers focus mostly on the human affinity for round numbers, with many finding significant price barriers at round numbers (Donaldson and Harold 1993, Sonnemans 2003, Koedijk and Stork 1994, ect.). Price barriers have been found by observing the frequency of specific stock prices and the rate of change in a stock’s price around those levels (Donaldson and Harold 1993, Sonnemans 2003). Humans seeing a change in the largest nonzero place holder (e.g. 19.9 changing to 20.0) as a larger jump than an equivalent monetary jump that leaves that number unchanged could be a psychological cause for price barriers in the stock market (Sonnemans 2003). The resistance of breaking a round number is found when a stock closes ending in a 9, as those stocks experience significantly higher levels of selling off than other stocks (Bagnoli et. al. 2006). There is also evidence of price barriers becoming levels of support as trading volume increases after a significant price level is broken (Donaldson and Harold 1993, Huddart 2005). The importance of price barriers in the stock market has been known to
traders for much longer than its discovery in academia. From the early works of W.D. Gann, and possibly before, some traders have attributed their profits to the knowledge of the proportionality of the stock market (Gann 1935).

Not all literature is as supportive of the importance of price barriers in the stock market, although most have found evidence of it to some extent. Emerging markets, possibly because of more rapid growth, do not exhibit strong support for the hypothesis of price barriers (Bahng 2003). Recently, even in more developed markets, such as some in Europe, price barriers were not found to be constant over time. Once the anomalies were recognized, they tended to disappear in accordance with the efficient market hypothesis (Dorfleitner and Klein 2009). Some find that while price barriers exist they are of no use to investors because knowledge of the barriers does not allow investors to predict a stock’s return (Koedijk and Stork 1994). Furthermore, automated investing could result in the formation of price barriers because limit orders are usually placed at round numbers, which would account for the clustering of prices and increase in trading volume when a stock price reaches a round number (Chiao and Wang 2009).

This paper examines the effect of price barriers in the stock market on the options market and attempts to identify price barriers in stock prices by anomalies in option pricing. If option prices take all market information into account, including price barriers, then the difference between the market price and the B-S predicted price would vary more near a barrier price. Examining price barriers’ effect on the B-S model extends beyond current literature on price barriers, which focuses mainly on locating barriers within stock markets. The paper combines two strands of literature by using similar methodology seen in previous studies on the systematic errors in the B-S model and looking for errors predicted by behavioral finance theories.
III. Methodology

This paper focuses on testing the validity of the Black-Scholes model when accounting for the presence of price barriers through the exploration of systematic errors. Previous literature has found price barriers at round numbers for multiple securities by looking at frequency of a security’s price (Donaldson and Harold 1993, Sonnemans 2003, Koedijk and Stork 1994, ect.). As the price of a stock approaches a price barrier the movement of the price slows, allowing for frequency analysis to locate the barriers. This results in the Brownian movement assumption of the B-S model being violated. The violation of this assumption at select price levels should create an error at those levels that is not seen otherwise. Evaluating the difference between the B-S predicted price and the market price is contingent on accurately evaluating the volatility of a security’s return to find the correct B-S price.

A. Finding the Correct B-S Price

To accomplish this initial task, we use the methodology of MacBeth and Merville in “An Empirical Examination of the Black-Scholes Call Option Pricing Model.” Their analysis is reliant on the assumption that the B-S model accurately predicts an at-the-money call option price. Black notes errors in his model on options far in- or out-of-the-money, and in options with less than 90 days to expiration (Black 1975). To account for this, the model used to estimate implied volatility of a security’s return controls for distance from the money using only options with greater than 90 days to expiration.

Taking the B-S implied volatility of an at-the-money option as the true volatility for the underlying security’s returns, we run a regression to estimate this volatility. The regression is run on all options traded on that day for a particular security. In total, 252 trading days per security, less the days the particular security
had fewer than 5 different options traded were used. The estimated regression, taken from MacBeth and Merville, is the observed implied volatility of an option’s market price regressed on its distance from the money. The model is given by

\[ \sigma_{ijt} = \theta_{i0t} + \theta_{i1t}m_{ijt} + \varepsilon_{ijt}, \]  

(4)

where \( i \) ranges from 1 to \( I \) representative of the \( I \) companies, \( t \) ranges from January 3, 2011 to December 30, 2011 for each trading day, and \( j \) ranges from 1 to \( J \), with \( J \geq 5 \), for all different options of company \( i \) on day \( t \). Here, the only control variable \( m \) is the distance the option is from the money as a percentage of the security’s price. More formally,

\[ m_{ijt} = \frac{s_{it} - X_{ijt}e^{-rT}}{s_{it}} \]  

(5)

where \( S \) is the stock price of company \( i \) on day \( t \), \( X \) is the strike price of option \( j \) of company \( i \) discounted by the risk free rate back to its present value. This measure is a slight variation on MacBeth and Merville’s work, where the difference is taken as a percentage of strike price. The use of call options to hedge positions caused their measure of distance from the money to be severely skewed, but this small variation decreases the skew substantially without drastically changing the results (Blyler 2012).

In the above regression, the intercept is the estimated implied volatility of an at-the-money option. Our assumption states that this estimate is the correct volatility of the underlying security’s return and should be used to find the B-S prediction of that security’s options at any strike price on the given day. This allows us to find the difference between the market price (\( C_{ijt} \)) and the B-S predicted price given by

\[ y_{ijt} = C_{ijt} - C_{BS}(\theta_{i0t}). \]  

(6)
In the above equation, is the market price of option \( j \) on day \( t \) of the underlying security and is the B-S predicted price of that call option using the estimated true volatility.

**B. Systematic Difference between Market Price and B-S Predicted Price**

MacBeth and Merville find the difference between the market price of a call option and the B-S predicted price to be a function of the distance of the option’s strike price from the money and the option’s time to expiration. MacBeth and Merville’s model estimating the difference between the market price and B-S predicted price of a call option is given by

\[
y_{ijt} = \alpha_0 + \alpha_1 m_{ijt} + \alpha_2 T_{ijt} + \varepsilon_{ijt} \quad (7)
\]

The regression is run separately, not only for each underlying security, but also for different properties of options. Based on previous literature, finding differences in pricing errors between options with short or long times to expiration and options in- or out-of-the-money, each underlying security has four separate regressions to allow different estimates for all possible combinations of options near or far from expiration and in- or out-of-the-money.

MacBeth and Merville’s model uses linear variables, but through empirical work a model including a squared term for distance from the money was found to be more appropriate (Blyler 2012). In their 1979 paper, the data is not treated as panel data although options are followed over time, resulting in heteroskedasticity and autocorrelation. More recent advances in panel data analysis have led to an increase in financial data being analyzed using panel techniques to correct for these problems inherent in the data (Petersen 2005, Gow et al. 2010). Previous authors using panel data and similar regression techniques to estimate volatility have suggested the square of distance from the money and the liquidity of both the underlying asset and option contribute to pricing
difference from the B-S option pricing model (Ncube 1996, Feng 2011). To allow analysis over all stocks, we introduce a new dependent variable,

\[ y_{ijt}^{\%} = \frac{y_{ijt}}{c_{ijt}} \]  

(8).

Therefore, MacBeth and Merville’s original model is revised so that

\[ j_t = \alpha_0 + \alpha_1 m_{ijt} + \alpha_2 m_{ijt}^2 + \alpha_3 \ln (vol)_{ijt} + \alpha_4 vol_{ijt} \cdot m_{ijt}^2 + \alpha_5 T_{ijt} + \alpha_6 PBA_{ijt} + \alpha_7 PBB_{ijt} + \epsilon_{ijt} \]  

(9).

The variables \( y, m, \) and \( T \) are taken from equation 7, and the addition of \( m^2 \) comes from our empirical work and other authors (Feng 2011, Blyler 2012). Furthermore, Feng’s work on the liquidity effect in the option market makes controlling for the volume of trading of each option on each day appropriate. In his study, volume of trading also affected the curvature of the error caused by distance from the money. In order to account for Feng’s finding, \( m \) is the number of trades an option had on a given day. Its effect on the curvature is controlled for by multiplying \( m \), while its overall effect on the error is given in log scale because of decreasing returns. The analysis of this paper focuses on the dummy variables \( y \) and \( m \), representing price barriers above and below the underlying security’s current price respectively. Price barriers above the current price, denoted \( y \), are expected to have a negative sign because they represent levels of resistance. Alternatively, price barriers below the current price, denoted \( m \), are expected to have a positive sign because they represent levels of support.

Previously mentioned price barriers have been found in the stock market by numerous authors; therefore, if the options market is efficient, then these price barriers should impact the market price of options because all available information is reflected in the price of the option. The B-S model assumption of Brownian motion in a security’s price does not allow for price barriers to exist. Hence, they cannot be priced into the B-S predicted price. If
price barriers in the stock market are priced into call options, then those options with strikes near the barriers should have differences between the market and B-S predicted price not explained by other independent variables. How price barriers affect a stock, and therefore the corresponding options price, differ depending on the location of the barrier relative to the security’s current price.

A price barrier above a security’s current price acts as a level of resistance to a price increase. As the price of a security approaches a level of resistance, its rate of change of price is expected to decrease. Furthermore, the probability of the price rising above the price barrier is less than the probability of it rising above an arbitrary level that offers no resistance. A smaller chance of breaking the level of resistance and an expected decrease in the rate of change should lower the price of options with strike prices near the barrier. Out-of-the-money options have a lower probability of finishing in-the-money if the strike price is at or slightly above the barrier. Additionally, finishing far enough in-the-money for the buyer to recoup the B-S predicted option premium is less likely if the strike price is slightly below the barrier.

Alternatively, a price barrier below a security’s current price can be seen as a level of support to a price drop. As the price of a security decreases towards a level of support, the absolute rate of change is expected to slow. The properties of a level of support act similarly to a level of resistance, but should result in a higher option price if the strike is near a level of support. Such an option is less likely to fall out-of-the-money before expiration and should command a higher premium from the buyer.
IV. Hypothesized Price Barriers

A. Bollinger Bands

In this section, possible locations for price barriers are presented along with hypothesized reasons and potential implications. The first location for possible price barriers comes from a commonly used financial technical indicator, Bollinger bands. Bollinger bands were made famous by analyst John Bollinger who believed “asking the market what is happening is always a better approach than telling it what to do” (Bollinger 1992). His standard for measuring volatility of a stock started with a simple $n$-period moving average of a stock’s price, which evolved into a weighted moving average in some cases, with $k$ standard deviations of the stock’s last $n$ prices added and subtracted from the moving average.

For our analysis the period length is Bollinger’s suggested 20 days and the distance from the simple moving average is two standard deviations (Bollinger 1992). Bollinger bands being a measure of historic volatility, an investor could view the bands as price barriers the security’s price is unlikely to break if historic estimates of volatility hold. She would then calculate the Bollinger bands at the end of a trading day and use those levels to adjust her valuation of options the upcoming day. Using this strategy, investors would buy options near the bottom Bollinger band and write options near the top Bollinger band, driving prices up and down respectively. Of course, an option’s strike price will rarely be exactly at a Bollinger band, meaning some distance must be deemed ‘close enough’ to the Bollinger band for investors to believe the effects of price barriers would play a role in evaluating the option.

Options have strike prices at $10 intervals if the underlying security’s price is greater than $200, $5 intervals if the security’s price is between $25 and $200, and $2.5 or $1 intervals if the price less than $25. Notice the strike price interval is never less than 5% of the underlying security’s price besides
the extreme cases of very expensive stocks. Working off of this, we deemed an option’s strike price ‘close enough’ to a Bollinger band if

\[|\text{Strike} - \text{Bollinger band}| < 0.025 \times \text{Underlying Security's Price}.\]

This method effectively creates a 5% interval around Bollinger band, but does not allow two different strike prices to be ‘close enough’ to the price barrier. Because of the uncertainty of the differences between options with strike prices directly above and strike prices directly below the price barrier, two separate dummy variables are utilized; however, the expected sign on both is the same depending on the strike being near the upper or lower Bollinger band.

Bollinger bands suffer from a few drawbacks that make them less likely price barriers than the other proposed barriers. Investors can use 10 period Bollinger bands as well as vary the number of standard deviations added and subtracted, which would result in differing opinions about the exact location of price barriers. Additionally, Bollinger bands change daily using this formula, meaning an investor using the above strategy would only believe the price barrier existed at that level for a day. Options affected by a barrier for a day would not vary greatly in worth as it is likely they do not expire for many days to come and the barrier would likely shift by then. A more realistic price barrier would stay constant throughout time, or at least for a meaningful length of time.

B. Round Numbers

Most current literature on price barriers focuses on round numbers, meaning integers when a security’s price is low, multiples of ten when the price is slightly higher, or multiples of 100 when the price is higher still. Many authors find the existence of price barriers at these numbers in a variety of securities and markets. Barriers at round numbers are the easiest to defend as given because the psychological reasoning behind their existence is a staple of behavioral finance.
Investors seem to weight round numbers more than their mathematical worth if markets are perfectly efficient.

Here the issue that arises is options are often sold at round numbers as explained above. To avoid this problem, barriers, or at least stronger barriers, are hypothesized to exist at round numbers seemingly more important to humans. For most securities, options are sold with strikes at multiples of ten or five because their underlying price is greater than $25. As such, for all securities with a price of $20 or more, the important round number strike prices are chosen to be at multiples of $50. For securities with a price less than $20, important round number strike prices were chosen at multiples of 5. While these numbers are reasonable selections, there is an inherent weakness is simply choosing these barriers rather than investigating all round numbers; however, our methodology would not hold if all round numbers were taken into account.

There are multiple price barriers at any given time and the arguments for round numbers result in barriers switching between support and resistance depending on their position relative to the security’s current price. This means a broken level of resistance becomes a level of support and should drive the price of the security higher as investors grow more confident in its performance. Similarly, a broken level of support becomes resistance and should drive the price down even further. This could counteract the differing value given to options with strikes near the barriers. A less risk adverse investor may be willing to pay more for an option with a strike near a level of resistance, knowing if the option moves into the money, it is likely to move further into the money. Additionally, an option with a strike near a level of support may not appear less risky knowing if the option moves out-of-the-money one day it is less likely to move back into the money than other options. The final hypothesized price barrier locations avoid both problems of changing over time and switching between support and resistance.
C. **Gann Levels**

Infamous investor W.D. Gann is known for his unorthodox beliefs about the stock market and the success he had with trading strategies based on seemingly unrelated occurrences. From the effects of planetary retrograde motion to the mathematical properties of geometric shapes, Gann’s trading methods are unconventional; however, numerous books have been written by Gann and others about these special proportions in markets (Brown 1999). While skepticism should accompany outlandish claims, the success and popularity of both the books and methods demonstrate many traders know about, if not use, Gann’s approach. Knowing investors in the stock market may act differently at special price levels, their expected actions should be priced into the options market. This makes no statement about the logic behind Gann’s method; rather, if enough investors believe his approach has merit it becomes a self-fulfilling prophecy.

Although Gann makes numerous claims, the analysis here will focus on levels created by a method referred to as Gann’s wheel. Gann’s wheel involves levels of support coming from a pivot high and levels of resistance coming from a pivot low where a pivot is an important price level. Following the methodology described in Brown 1999, we examined the charts of each stock in 2010 to find initial pivot highs and lows of our stocks. These chosen pivots remained until the stock’s price fell below the pivot low or rose above the pivot high, at which point the broken pivot switched to the new year to date high or low. From here, Gann describes the angles 45, 90, 120, 180, 240, 270, 315, and 360 degrees of having particular importance. In a Gann wheel, these angles are drawn from the pivot low and pivot high on a plane of price and time until they intersect what Gann calls the square of nine. The time of the intersection is Gann’s prediction for when

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the stock’s price will reach that barrier; however for this analysis we will only focus on the price level at which the intersection occurs. An example of these intersections can be seen in the figure below.

(Brown pg 234, 1999)

These price barriers will remain constant until either the pivot high or pivot low changes, and the important intersections can be calculated by the following equations for a given degree.

Resistance level for degree \( d \) 

\[
(\sqrt{\text{Pivot Low}} + \frac{d}{180})^2
\]  

(10)

Support level for degree \( d \) 

\[
(\sqrt{\text{Pivot High}} - \frac{d}{180})^2
\]  

(11)

Similar to the Bollinger Band price barriers, options are unlikely to be sold with strike prices exactly at Gann’s levels. Again, the strike price is deemed ‘close enough’ to the price barrier if

\[
|\text{Strike} - \text{Gann Level}| < 0.025 \times \text{Underlying Security's Price},
\]

creating a buffer area for the strike price to fall.
The drawbacks of Gann’s price barriers come from their complexity and often misunderstood fundamentals. Different trading programs calculate Gann levels with built in functions; however, these functions differ in their implementations of Gann’s wheel (Brown 1999). If investors are not in agreement on the location of Gann’s price barriers, the effect on option prices would not be significant. This could allow traders who correctly estimate Gann’s levels to make larger gains or it could push investors away from Gann’s method towards a more straightforward method.

V. Data

The data for this paper is called National Best Bid Offer (NBBO) data collected by Options Pricing and Reporting Authority (OPRA). It contains information on the traded call options of 33 companies, most of which currently make up the DOW 30, as well as several important technology firms. The firms and their ticker symbols are shown in the following table.

<table>
<thead>
<tr>
<th>Company Name</th>
<th>Ticker Symbol</th>
<th>Company Name</th>
<th>Ticker Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple</td>
<td>AAPL</td>
<td>International Business Co.</td>
<td>IBM</td>
</tr>
<tr>
<td>American Express Co.</td>
<td>AXP</td>
<td>Johnson &amp; Johnson</td>
<td>JNJ</td>
</tr>
<tr>
<td>Bank of America Corp.</td>
<td>BAC</td>
<td>JP Morgan Chase and Co.</td>
<td>JPM</td>
</tr>
<tr>
<td>Boeing Co.</td>
<td>BAC</td>
<td>The Coca-Cola Co.</td>
<td>KO</td>
</tr>
<tr>
<td>Citigroup Inc.</td>
<td>C</td>
<td>3M Company</td>
<td>MMM</td>
</tr>
<tr>
<td>Caterpillar Inc.</td>
<td>CAT</td>
<td>McDonald's Corp.</td>
<td>MCD</td>
</tr>
<tr>
<td>Cummings Inc.</td>
<td>CMI</td>
<td>Microsoft Corp.</td>
<td>MSFT</td>
</tr>
<tr>
<td>Chevron Corp.</td>
<td>CVX</td>
<td>Pfizer Inc.</td>
<td>PFE</td>
</tr>
<tr>
<td>E.I. du Pont de Nemours and Co.</td>
<td>DD</td>
<td>Qualcomm Inc.</td>
<td>QCOM</td>
</tr>
<tr>
<td>Walt Disney Co.</td>
<td>DIS</td>
<td>Transocean Ltd.</td>
<td>RIG</td>
</tr>
<tr>
<td>General Elective Co.</td>
<td>GE</td>
<td>AT&amp;T Inc.</td>
<td>T</td>
</tr>
<tr>
<td>Google Inc.</td>
<td>GOOG</td>
<td>United Technologies Corp.</td>
<td>UTX</td>
</tr>
<tr>
<td>The Goldman Sachs Group, Inc.</td>
<td>GS</td>
<td>Verizon Communications Inc.</td>
<td>VZ</td>
</tr>
<tr>
<td>Halliburton Co.</td>
<td>HAL</td>
<td>Wal-Mart Stores Inc.</td>
<td>WMT</td>
</tr>
<tr>
<td>Hewlett-Packard Co.</td>
<td>HPQ</td>
<td>Exxon Mobil Corp.</td>
<td>XOM</td>
</tr>
<tr>
<td>Home Depot Inc.</td>
<td>HD</td>
<td>Standard &amp; Poors 500</td>
<td>SPY</td>
</tr>
<tr>
<td>Intel Corp.</td>
<td>INTC</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The data is end of day data from all 252 trading days in 2011 with strike prices ranging from far in-the-money to far out-of-the-money. The times to expirations at the initial sale of the options include one month, three months, six months, and a few times of a year or more. All options are followed until expiration and every day includes the ending spreads, greeks, and trading volumes for the options.

Since the analysis relies on market prices containing all information, only options traded (trading volume greater than zero) on a specific day are included in the analysis. Because the companies involved in the analysis are commonly traded companies, this does not eliminate many options besides those with strike prices too far away from the underlying security’s price to be useful to traders, or those with extremely long times to expiration. In order to eliminate data we viewed as likely mistakes, options that were sold for less than the intrinsic value of the option were not included. Additionally, options with less than two days to expiration are not included in the analysis because end of the day data is not reflective of the changes occurring with only a couple days left in the life of the option.

Using regressions to estimate the true volatility of a stock’s return creates a weighted average of implied volatility from all of the traded options on that day. In order to assure enough inputs into the weighted average, the true implied volatility is only calculated if options with 5 different strikes or expirations were traded that day. This estimate is then used to calculate the inputs required in the Black-Scholes option pricing model. In order to estimate the annualized risk free rate, the annualized return rate on a three month U.S. Treasury Bill was obtained from the St. Louis Federal Reserve. The time to expiration is obviously not always three months; however, this should not be a problem because annualized Treasury Bill returns only vary slightly with changes in time to maturity from a day to a
year, and the B-S predicted call price lacks sensitivity to the riskless return rate. In fact, MacBeth and Merville note, “…[their] results would be virtually identical had [they] used a single riskless return for a Treasury Bill… .”

The data used differs from MacBeth’s and Merville’s data in two distinct ways. First, the data includes information on more than five times the number of stocks. Second, many days have over fifty observations used in the regression to calculate the estimated at-the-money volatility. These advantages allow for stronger conclusions because it reduces the likelihood that small sample sizes will cause the results.

VI. Results

Looking at the difference between the market price and B-S predicted price as a percent of the market price allows for comparison of options regardless of the underlying stock. However, our data set does not allow for comparison regardless of distance from the money. Using our formula the percent error cannot go above 100%, but it has no lower bound. When options are in-the-money, $y\%$ has a smaller range and a reasonable standard deviation, with summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observations</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y%$</td>
<td>296554</td>
<td>0.534</td>
<td>12.091</td>
<td>-2622</td>
<td>98</td>
</tr>
</tbody>
</table>

On the other hand, with options out of the money, $y\%$ has a much larger range, as seen below:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observations</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y%$</td>
<td>398889</td>
<td>-61.05</td>
<td>154</td>
<td>-16620</td>
<td>100</td>
</tr>
</tbody>
</table>

The extreme difference leads to drastically different coefficient estimates in our regression, which are divided up between in- or out-of-the-money and less or greater than 90 days to expiration. This division is based on the differences in pricing observed by Black, MacBeth and Merville, and others. As such, the
coefficients of the regressions are less important than the significance and sign of the coefficients. In all regressions, observations are demeaned by the underlying stock and the standard errors are clustered by option, allowing for correlation between the errors of the same option sold on different days. Clustering by option, prevents the autocorrelation that plagued previous papers (see MacBeth and Merville 1979, Blyler 2012) and makes the significance tests useable.

Running the regression on options in the money with more than 90 days to expiration yields the following results where positive coefficients mean higher relative market value.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observations</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>y%</td>
<td>296554</td>
<td>0.534</td>
<td>12.091</td>
<td>-2622</td>
<td>98</td>
</tr>
<tr>
<td>y%</td>
<td>398889</td>
<td>-61.05</td>
<td>154</td>
<td>-16620</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Month</th>
<th>Observations</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>m'</td>
<td>148696</td>
<td>28.032**</td>
<td>(1.123)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m'^2</td>
<td>148696</td>
<td>-34.192**</td>
<td>(2.293)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume</td>
<td>163745</td>
<td>27.899**</td>
<td>(1.318)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume * m'^2</td>
<td>148696</td>
<td>-33.014**</td>
<td>(2.286)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower BB Above</td>
<td></td>
<td>-0.152**</td>
<td>(0.011)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower BB Under</td>
<td></td>
<td>0.373**</td>
<td>(0.104)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gann Support Above</td>
<td></td>
<td>0.051</td>
<td>(0.085)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gann Support Below</td>
<td></td>
<td>0.124</td>
<td>(0.082)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round Number Under</td>
<td></td>
<td>0.114</td>
<td>(0.180)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>148696</td>
<td>-3.060**</td>
<td>(0.202)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>148696</td>
<td>-3.053**</td>
<td>(0.200)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td></td>
<td>-3.468**</td>
<td>(0.214)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Robust standard errors in parentheses |
+ significant at 10%; * significant at 5%; ** significant at 1% |

The first proposed price barriers, Bollinger bands, have contradicting signs for their coefficients; however, only the coefficient on the variable for options with strikes slightly below the lower Bollinger band is statistically significant. This is promising as it has the sign predicted by our hypothesis that
investors likely view the lower Bollinger band as a level of support and therefore options with strikes slightly below the band are more likely to finish in the money and should have a higher market value than other equivalent options. The results for options with strikes at our selected round numbers also suggest investors may view round numbers as a level of support, but the coefficient is not statistically significant. An option with a strike near a Gann level of support, while yielding coefficients with the sign predicted by our hypothesis, also does not result in a statistically significant increase in the option’s price. While the results of the first regression are promising, similar results are needed in order to provide strong evidence for our hypothesis.

The same regressions run on options with less than 90 days to expiration yields certain contradictory results, preventing stronger inferences from being made about levels of support.

<table>
<thead>
<tr>
<th></th>
<th>y%</th>
<th>y%</th>
<th>y%</th>
</tr>
</thead>
<tbody>
<tr>
<td>m’</td>
<td>32.181**</td>
<td>32.228**</td>
<td>30.334**</td>
</tr>
<tr>
<td></td>
<td>(1.633)</td>
<td>(1.621)</td>
<td>(2.248)</td>
</tr>
<tr>
<td>m’^2</td>
<td>-54.963**</td>
<td>-55.247**</td>
<td>-52.348**</td>
</tr>
<tr>
<td></td>
<td>(3.288)</td>
<td>(3.297)</td>
<td>(3.315)</td>
</tr>
<tr>
<td>month</td>
<td>-0.693**</td>
<td>-0.698**</td>
<td>-0.823**</td>
</tr>
<tr>
<td></td>
<td>(0.059)</td>
<td>(0.059)</td>
<td>(0.082)</td>
</tr>
<tr>
<td>volume</td>
<td>-0.551**</td>
<td>-0.560**</td>
<td>-0.742**</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.035)</td>
<td>(0.053)</td>
</tr>
<tr>
<td>volume * m’^2</td>
<td>0.077**</td>
<td>0.080**</td>
<td>0.177**</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.023)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>Lower BB Above</td>
<td>-0.498**</td>
<td></td>
<td>-0.455**</td>
</tr>
<tr>
<td></td>
<td>(0.170)</td>
<td></td>
<td>(0.163)</td>
</tr>
<tr>
<td>Lower BB Below</td>
<td>0.496**</td>
<td></td>
<td>-0.044</td>
</tr>
<tr>
<td></td>
<td>(0.137)</td>
<td></td>
<td>(0.153)</td>
</tr>
<tr>
<td>Gann Support Above</td>
<td></td>
<td>-0.455**</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.163)</td>
<td></td>
</tr>
<tr>
<td>Gann Support Below</td>
<td></td>
<td>-0.044</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.153)</td>
<td></td>
</tr>
<tr>
<td>Round Number Under</td>
<td>0.279</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.245)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-2.689**</td>
<td>-2.749**</td>
<td>-3.312**</td>
</tr>
<tr>
<td></td>
<td>(0.246)</td>
<td>(0.237)</td>
<td>(0.407)</td>
</tr>
<tr>
<td>Observations</td>
<td>111662</td>
<td>111662</td>
<td>132823</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.15</td>
<td>0.15</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Switching from options that are far from expiration to options that are near to expiration not only affected the signs of some of our variables of interest but also their significance levels. Options with strikes just below the lower Bollinger Band once again have an increased value in the market’s view and this increase is statistically significant. Once again, options with strikes slightly above the lower Bollinger Band are less valuable relative to the B-S predicted price, but this time the difference is statistically significant. Our hypothesis has no explanation for this phenomenon, but the positive and statistically significant coefficient when the strike is slightly below the lower Bollinger band provides evidence that investors do use the previous day’s Bollinger band to predict the stock’s future movement and investors believe a stock’s price is less likely to fall below the lower Bollinger band than an arbitrarily selected price. A possible reason for this phenomenon is the liquidity of the options market allows investors to trade frequently; therefore, they only try to predict movements for a short time before selling the option.

Price barriers at round numbers appear to have the expected affect in the options market, although the coefficients were not statistically significant in either regression. Price barriers being a relatively new discovery in the stock market, investors may be slow to change their evaluation methods. A smaller number of investors using round numbers as levels of support may be pushing the price of these options higher, but not a significant amount because other investors continue to pull the price downwards.

The result from Gann’s levels of support in this second regression is most surprising. The statistically significant negative sign suggests investors pay attention to Gann’s levels; however, investors value options with strikes near Gann’s levels of support less than other options. The inconsistency between options near to expiration and options far from expiration may be from the time aspect of Gann’s wheel that is not incorporated into this analysis. If Gann’s wheel
predicts a level of support being reached long after an option expires, investors are unlikely to put weight into Gann’s analysis. Our hypothesis does not explain the statistically significant negative coefficient for options with strikes near Gann’s level of support with less than 90 days to expiration. It does not appear that Gann’s levels of support, at least using the levels our methodology located, are of importance to investors.

Next, the regressions are run on options out-of-the-money; thus we move from level of support to resistance. These coefficients are not comparable to the previous two tables; however, the signs and significance test are still viable. We again begin with options with more than 90 days to expiration and find the following with regards to levels of resistance.

<table>
<thead>
<tr>
<th>m'</th>
<th>y%</th>
<th>y%</th>
<th>y%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>374.262**</td>
<td>379.530**</td>
<td>358.130**</td>
</tr>
<tr>
<td></td>
<td>(14.297)</td>
<td>(13.867)</td>
<td>(12.802)</td>
</tr>
<tr>
<td>m'^2</td>
<td>90.057**</td>
<td>92.373**</td>
<td>85.137**</td>
</tr>
<tr>
<td></td>
<td>(11.765)</td>
<td>(11.922)</td>
<td>(11.329)</td>
</tr>
<tr>
<td>month</td>
<td>3.035**</td>
<td>3.062**</td>
<td>3.091**</td>
</tr>
<tr>
<td></td>
<td>(0.191)</td>
<td>(0.195)</td>
<td>(0.176)</td>
</tr>
<tr>
<td>volume</td>
<td>1.110**</td>
<td>1.247**</td>
<td>1.090**</td>
</tr>
<tr>
<td></td>
<td>(0.326)</td>
<td>(0.341)</td>
<td>(0.314)</td>
</tr>
<tr>
<td>volume * m'^2</td>
<td>0.313</td>
<td>0.314</td>
<td>0.343</td>
</tr>
<tr>
<td></td>
<td>(0.300)</td>
<td>(0.301)</td>
<td>(0.304)</td>
</tr>
</tbody>
</table>

Upper BB Above
7.888**
(1.319)

Upper BB Below
6.609**
(1.550)

Gann Resistance Above
11.594**
(1.582)

Gann Resistance Below
16.573**
(1.722)

Round Number Above
-4.877
(3.395)

Constant
-14.314**
(2.321)

Observations
220929
220929
237111

R-squared
0.27
0.27
0.28
Analyzing out-of-the-money options with strikes near Bollinger Bands, the results are opposite of what our hypothesis suggests. Options with strikes around the upper Bollinger band appear to have a higher relative value to investors. A possible explanation is investors believe the upper Bollinger Band acts as a spring board and once broken investors will buy up the stock, increasing the price significantly past the strike price. As seen in options out-of-the-money, investors seem to impute a higher probability to a stock’s price staying within the Bollinger bands, so the increase in price expected if the upper Bollinger band is broken must be large enough to offset this decreased probability of its occurrence. Alternatively, investors may simply put less weight in the upper Bollinger band, believing the market will be bullish and the upper Bollinger band will not be as important.

Options with strikes at round numbers continue demonstrate the expected effects of a price barrier on the option market; however, the coefficient is still not statistically significant. Gann’s levels of resistance on the other hand, display similar traits to the upper Bollinger band. Options with strikes near to a Gann level of resistance, whether the strike price is slightly above or below the barrier, have a higher relative value to investors. Since Gann levels do not switch between support and resistance relative to the underlying stock’s price, our hypothesis does not explain this occurrence.

Similarly, the regressions on options close to expiration find the following results, supporting the results for levels of resistance from options far from expiration.
In options near to expiration, the only proposed price barrier with a significant effect is the Bollinger band. Again, options with strikes near the upper Bollinger band are of relative higher value to investors, contradicting our hypothesis. The Gann barriers appear to be insignificant when the option is close to expiration for both levels of support and resistance. This is most likely explained by our exclusion of the time component of Gann’s wheel.

Options with strikes near our selected important round numbers again have the expected devaluation associated with a level of resistance. Throughout our analysis, round numbers have not had a statistically significant coefficient, but have continually had the predicted change relative to the B-S predicted price.

Overall, the results for round numbers and the lower Bollinger band are promising, but the significant coefficients of the wrong sign do not provide strong support for our hypothesis. Because Bollinger bands and Gann levels have
not been explored as price barriers in the literature, it is not surprising options with strikes near those levels do not act as expected; although, the statistically significant coefficients suggests more research is needed at these levels. The obvious skew caused by taking the difference as a percentage of market price hinders comparison and most literature focuses on absolute difference between the market price and the B-S price. Further exploration into the effect of examining the error as a percentage of market prices is needed before absolute conclusions can be drawn.

VII. Conclusions

A. Price Barrier Hypothesis

We propose three price barriers, Bollinger bands, round numbers, and Gann levels, in the stock market and attempt to find evidence of their internalization in the options market. Options with strikes near levels of support are expected to have higher market prices, whereas the opposite should be true if the strikes are near a level of resistance because the B-S price does not take the lowered probability of breaking a price barrier into account. To find relatively lower and higher prices, the market price is compared to the Black-Scholes Option Pricing Model predicted price, which does not allow for price barriers to exist in the stock market.

We find evidence of systematic deviation from the B-S price at Bollinger bands in options both in- and out-of-the-money. Gann levels of resistance had the opposite effect of what was expected, and the levels of support did not appear to have an effect on the price of options. This suggests investors are more comfortable with conventional measures when attempting to estimate volatility; however, if Gann levels act as price barriers in the stock market, investors could make substantial profits by buying options at levels of support and selling those at
levels of resistance. Testing this proposed trading method is left for future work.

The results for options with strikes near round numbers were inconclusive although there was evidence of them beginning to be treated as levels of support and resistance. Although no coefficient was statistically significant, all had the sign predicted by our hypothesis. Not only does this support the behavioral finance findings of price barriers in the stock market, but also shows efficiency in the options market as option traders recognize patterns in the stock market not allowed by the efficient market hypothesis.

Closer to hypothesized results were found when analyzing options in-the-money, possibly because of the use of options as a hedge. Increased attention to in-the-money options could push the price closer to the fair present value, while out-of-the-money options have a less efficient price. This paper does not attempt to prove the existence of price barriers in the stock market; rather it examines how the options market acts around special price values. Significant results provide evidence that investors do care about certain price levels more than others, although further exploration is needed to completely understand how the option market internalizes investor preferences of these numbers. However, when looking for systematic errors in the B-S model, special price levels should be included with the commonly accepted distance from the money, time to expiration, and liquidity control variables. The discovery of price barriers affecting the options market allows future research on price barriers to occur regardless of a stock’s current price.

B. Updated Expected Volatility Hypothesis

Differences between the market price and B-S predicted price of options with strikes near Bollinger bands and Gann levels had differences not explained by previously found control variables, but these differences were not the differences predicted by our hypothesis. The coefficients on the dummy variables being
statistically significant indicate investors do pay attention to these levels, but not in our hypothesized manner. When the Bollinger bands were below the current price, hypothesized to act as levels of support, options just above (inside) the band were valued less, while options just below (outside) the band were valued more. For both Bollinger bands and Gann levels, when the strike was near those proposed barriers and above the strike price the results were opposite the hypothesized result. If those barriers acted as levels of resistance, the coefficient would be negative, but in both cases it was positive and statistically significant.

To explain these divergences from our original hypothesis, we propose an alternative explanation. Bollinger bands and Gann levels may not have any effect on a stock’s price, but investors may pay attention to them for other reasons. The B-S predicted price, which is commonly used by investors as a baseline, depends heavily on expected volatility of a stock. When evaluating a stock, investors often look at not only the price, but also technicals such as Bollinger bands. Therefore, an investor may use Bollinger bands and Gann levels as indicators their original estimation of volatility needs to be updated.3 Updating expected volatility around these levels would result in higher values for the option relative to the B-S price that holds a constant expected volatility estimate. This hypothesis explains the positive coefficient around barriers proposed to be levels of resistance and why levels of support followed our initial hypothesis more closely than resistance. Further research is needed to test the validity of this hypothesis, but it appears as if academic researchers need to obtain more information from real investors before price barriers can be fully explored.

---

3 Imagine seeing a stock price with Bollinger bands and using its historical volatility to estimate volatility. Since Bollinger bands are 2 SD away from the moving average, if the price breaks out of the bands, then the historical estimate is likely incorrect. At that time it may be best to update expected volatility to a larger value. Therefore, if Bollinger bands or Gann levels are reached, expected volatility expands and options are worth more.
VIII. References


